DISPEC Training School

Spatiotemporal Characterization of the Ionosphere

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Outline

1. Singular Value Decomposition
2. SVD on GIMs
3. Higher Order SVD
4. HOSVD on GIMovies

Spatiotemporal Characterization. SVD

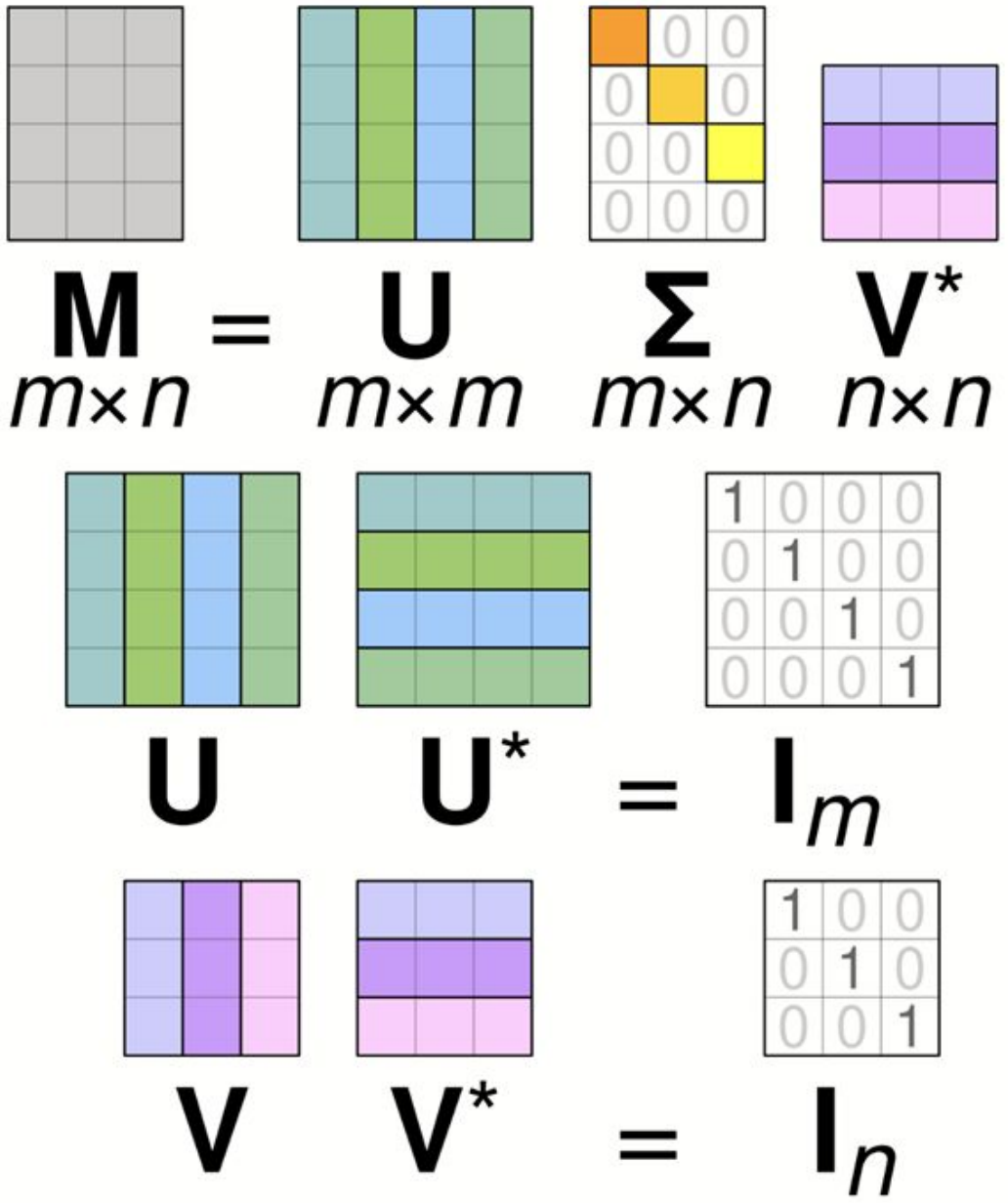
Singular Value Decomposition

SVD is factorization method for matrices.

Think of it as a generalization of diagonalization of matrices.

U columns and V rows form orthonormal bases.

Σ contains singular values along the diagonal.



Spatiotemporal Characterization. SVD

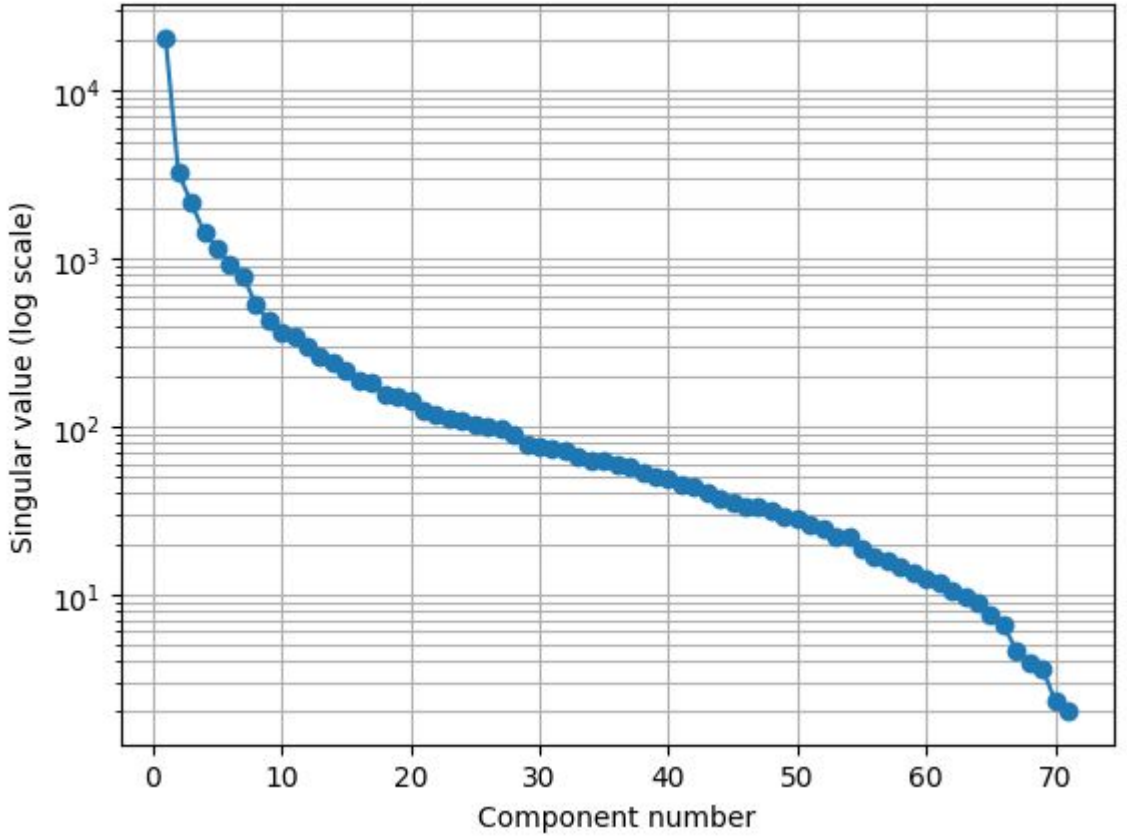
SVD. Data Compression

$$M = U\Sigma V^T = \sum_{i=1}^{p=min(m,n)} \sigma_i\left(\vec{u}_i \otimes \vec{v}_i^T\right)$$

SVD Hierarchy → $\sigma_1 \geq \sigma_2 \ldots \geq \sigma_{1p} \geq 0$

If $\sigma_1 \gg \sigma_{l<m,n}$, then we can drop out some terms. → **Data Compression**

$$M \approx \sum_{i=1}^{p<min(m,n)} \sigma_i\left(\vec{u}_i \otimes \vec{v}_i^T\right)$$



Example of SVD hierarchy on a VTEC GIM.

Spatiotemporal Characterization. SVD

SVD. Pattern characterization.

If $\sigma_1 \gg \sigma_{l<m,n}$, then the matrix is almost equivalent to a few structures/patterns

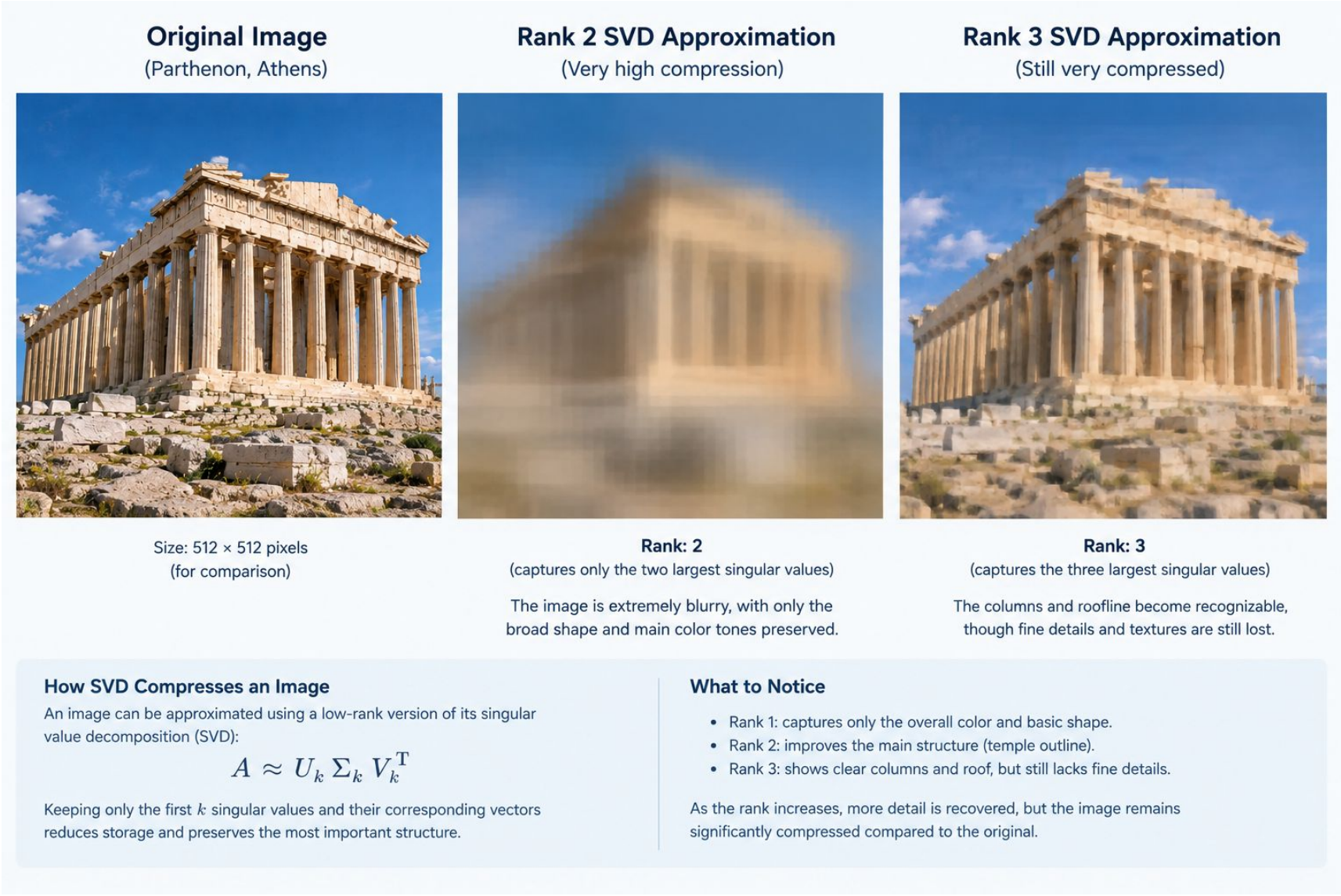
$$M \approx \sum_{i=1}^{p<min(m,n)} \sigma_i \left(\vec{u}_i \otimes \vec{v}_i^T\right)$$

Spatiotemporal Characterization. SVD

SVD. Pattern characterization.

The original image has 512x512 pixels.
 Up to 512 singular values.

The image at rank 20 is almost identical to the original.



Spatiotemporal Characterization. SVD

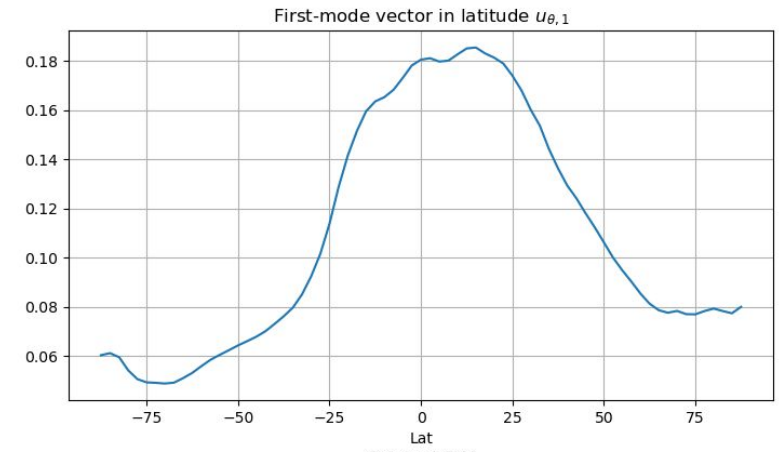
SVD. Pattern characterization. VTEC GIMs

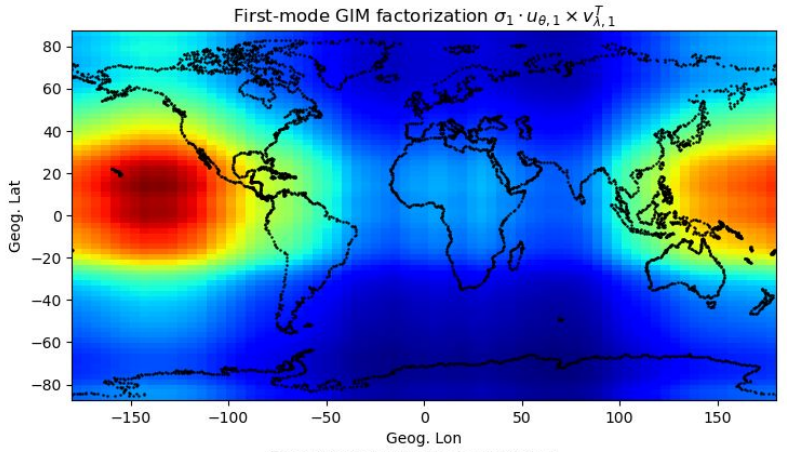
GIMs are images of 71x73 pixels size.→ SVD decomposition provides information about the main structures in space.

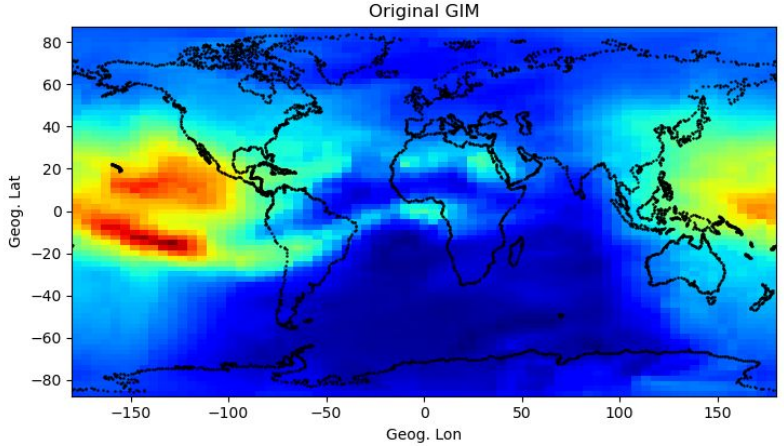
$$M_{GIM} = \sum_{i=1}^{71} \sigma_i \left(\vec{u}_i \otimes \vec{v}_i^T\right)$$

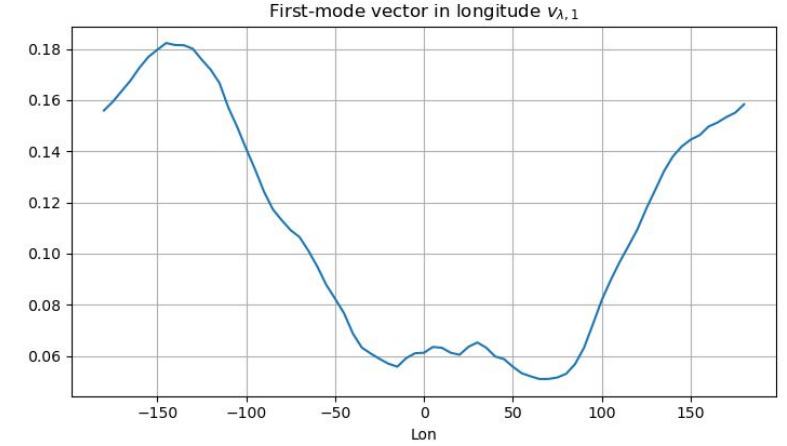
$$Latitude\ Orthonormal\ Base\ \{\vec{u}_1..\vec{u}_{71}\}$$
$$Longitude\ Orthonormal\ Base\ \{\vec{v}_1..\vec{v}_{71}\}$$

$$M_{1^{st}} = \sigma_1 (\vec{u}_1 \otimes \vec{v}_1^T)$$









The first-mode is characterized by the Equator Ionospheric Anomaly.

Spatiotemporal Characterization. SVD

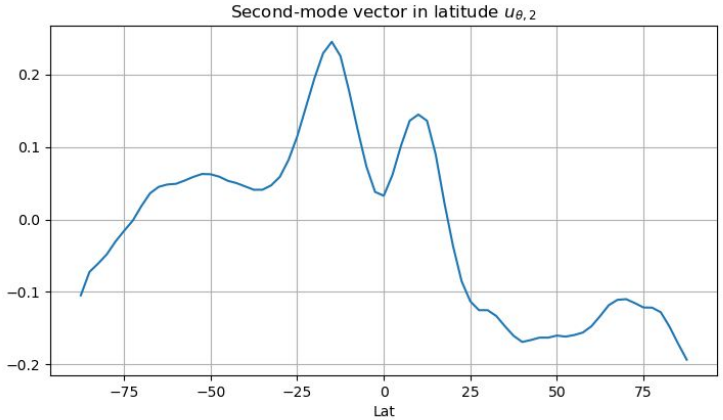
SVD. Pattern characterization. VTEC GIMs

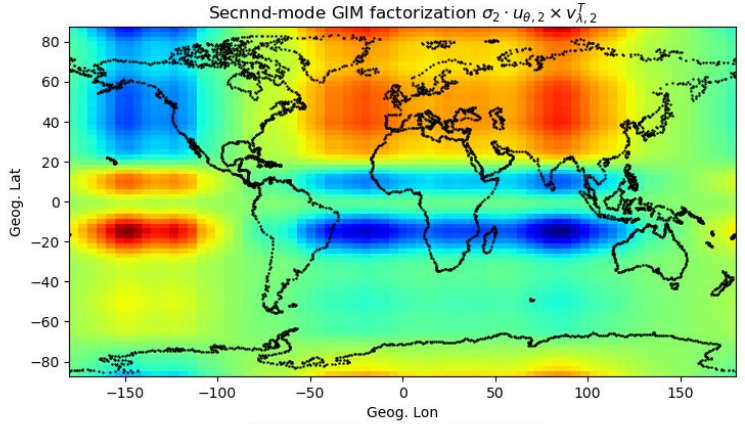
GIMs are images of 71x73 pixels size.→ SVD decomposition provides information about the main structures in space.

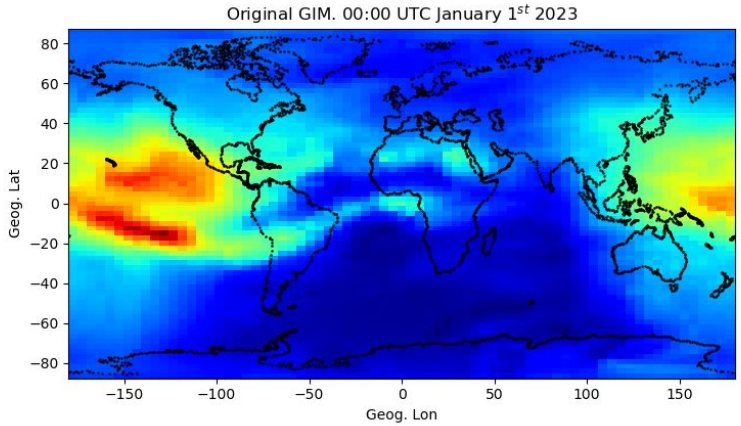
$$M_{GIM} = \sum_{i=1}^{71} \sigma_i \left(\vec{u}_i \otimes \vec{v}_i^T\right)$$

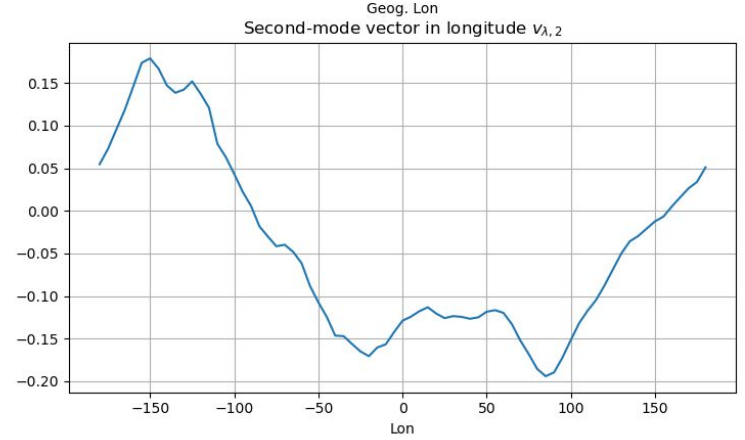
$$Latitude\ Orthonormal\ Base\ \{\vec{u}_1..\vec{u}_{71}\}$$
$$Longitude\ Orthonormal\ Base\ \{\vec{v}_1..\vec{v}_{71}\}$$

$$M_{2nd} = \sigma_2(\vec{u}_2 \otimes \vec{v}_2^T)$$







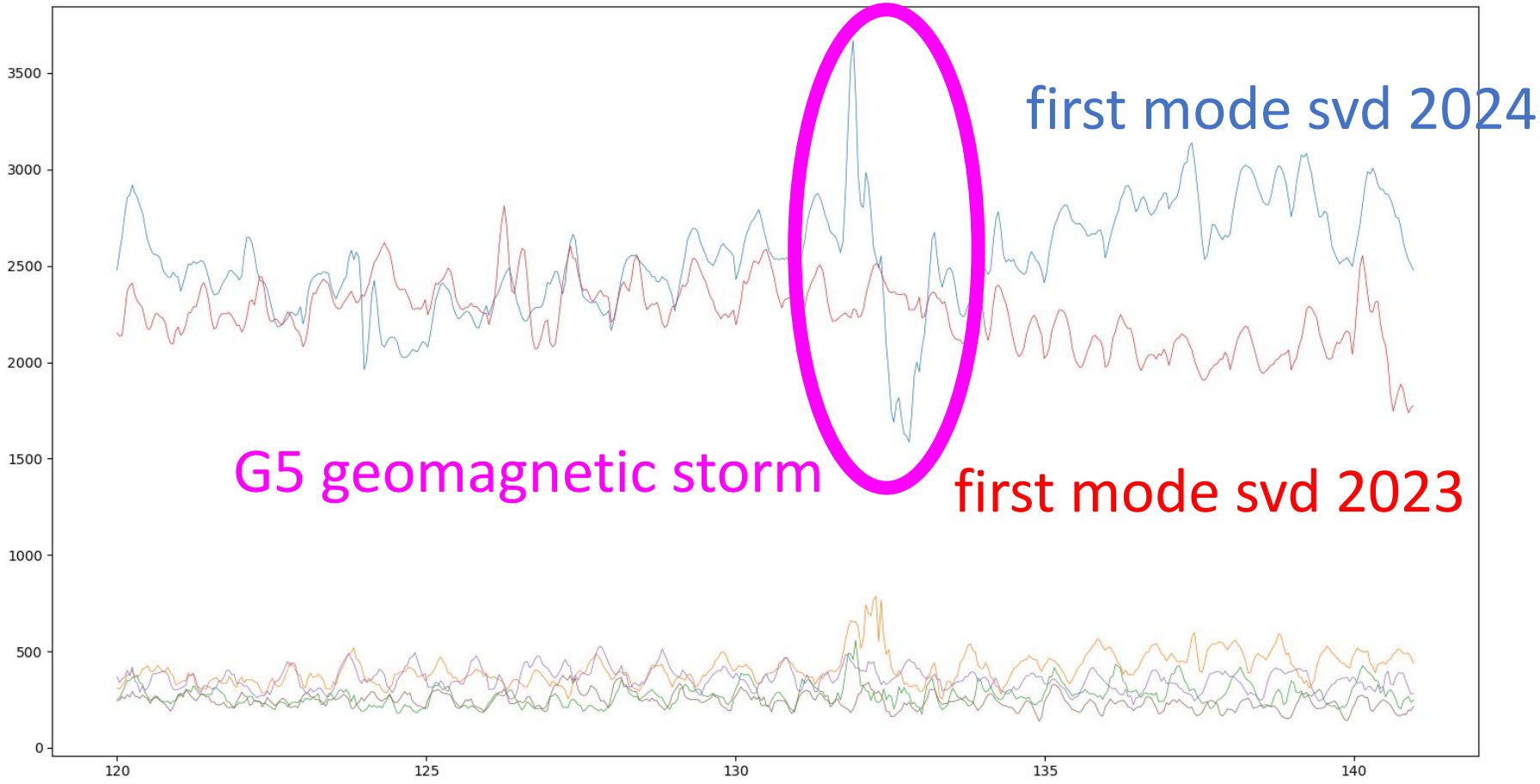


The second-mode shows the asymetry between hemispheres and the two-peaks shape of the fountain effect in the EIA.

Spatiotemporal Characterization. SVD

SVD. Pattern characterization. Singular Values Time Series

SV σ_i time series for the period [120, 140] doys in 2023 (red-top) and 2024 (blue-top).



Spatiotemporal Characterization. HOSVD

Higher Order SVD. VTEC GIMovies

Bases for Time (chrono-vectors), Longitude and Latitude.
The following example uses Geographic Longitude and Geomagnetic Latitude.

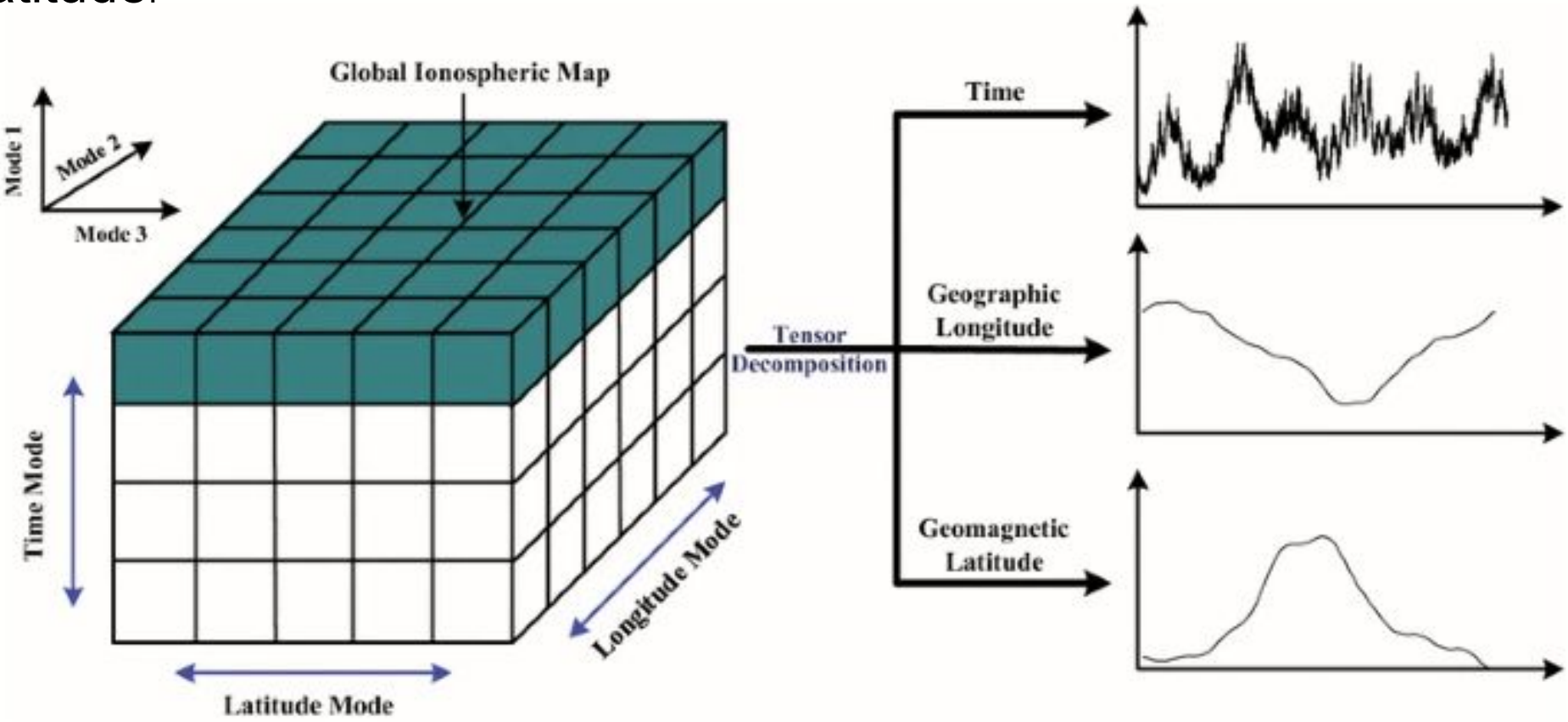
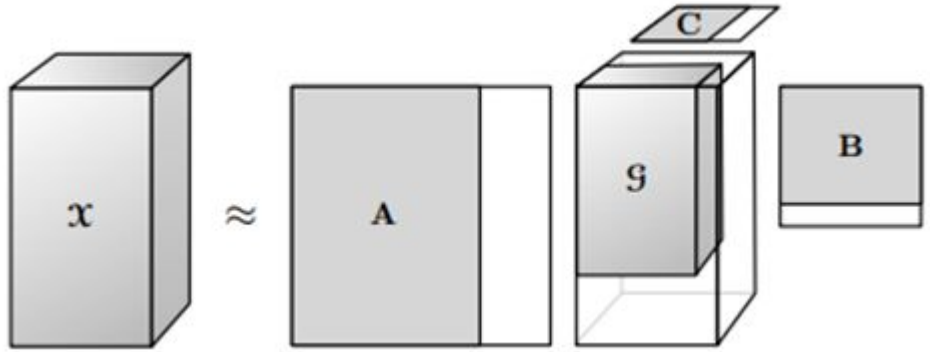


Image from Lu et al. (2018)

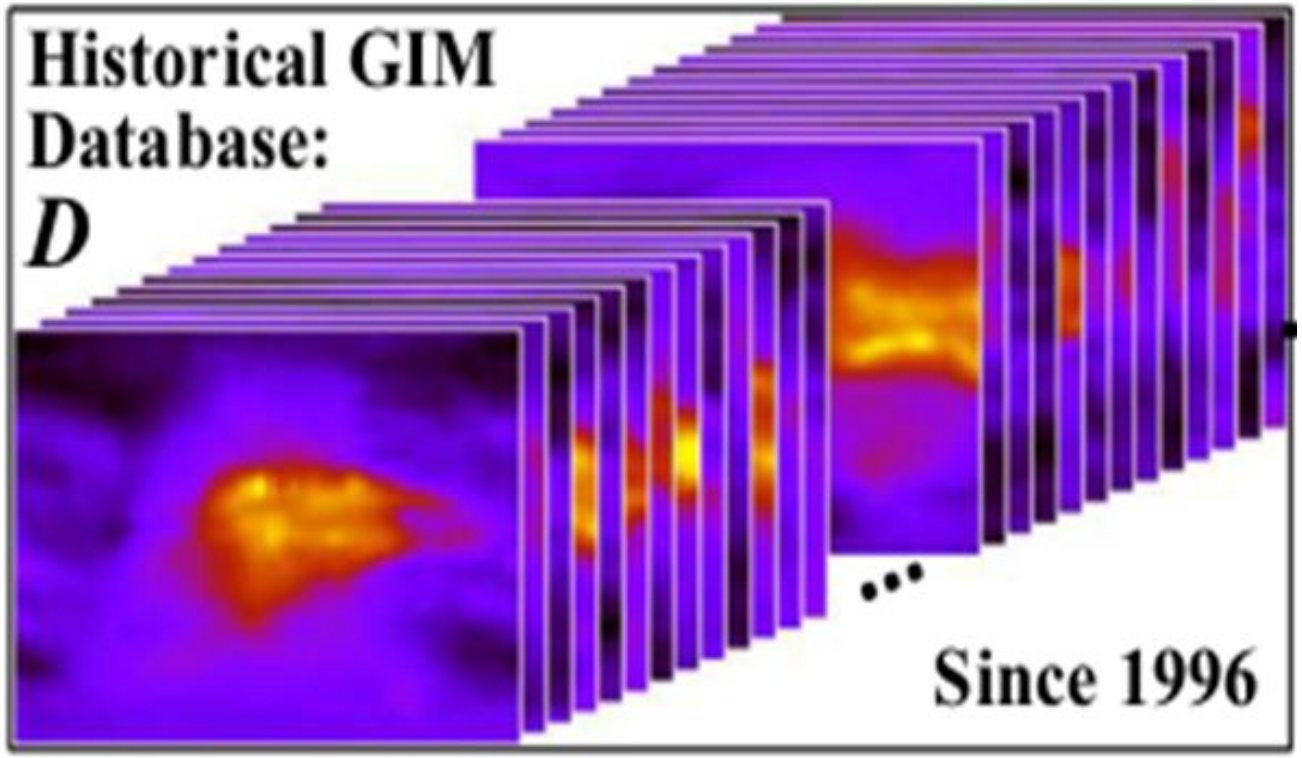
Spatiotemporal Characterization. HOSVD

Higher Order SVD



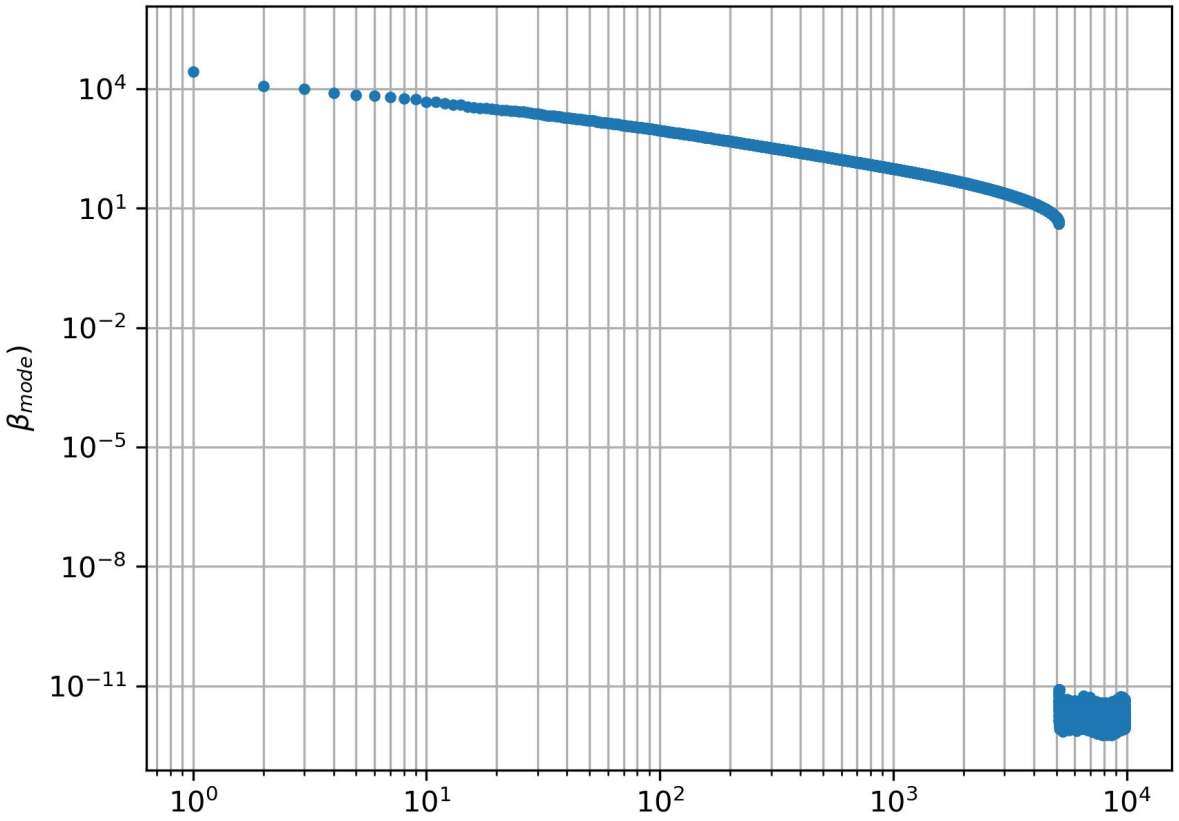
Columns in A, B and C matrices form orthonormal bases.

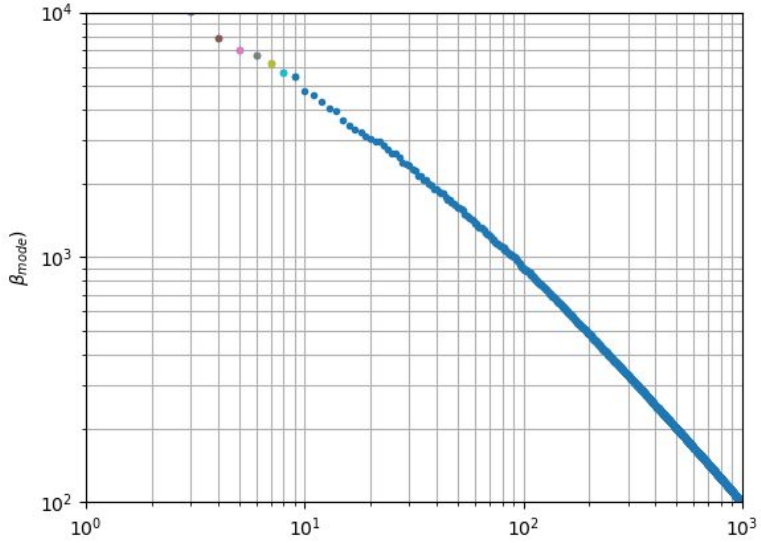
Core tensor G contains “singular values” along and *outside* the diagonal.

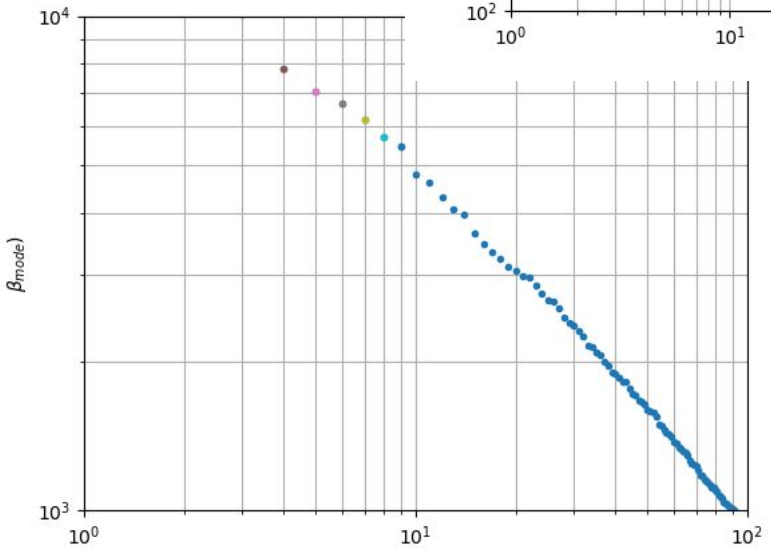


Spatiotemporal Characterization. HOSVD

Higher Order SVD. Core tensor coefficients β or “singular values”

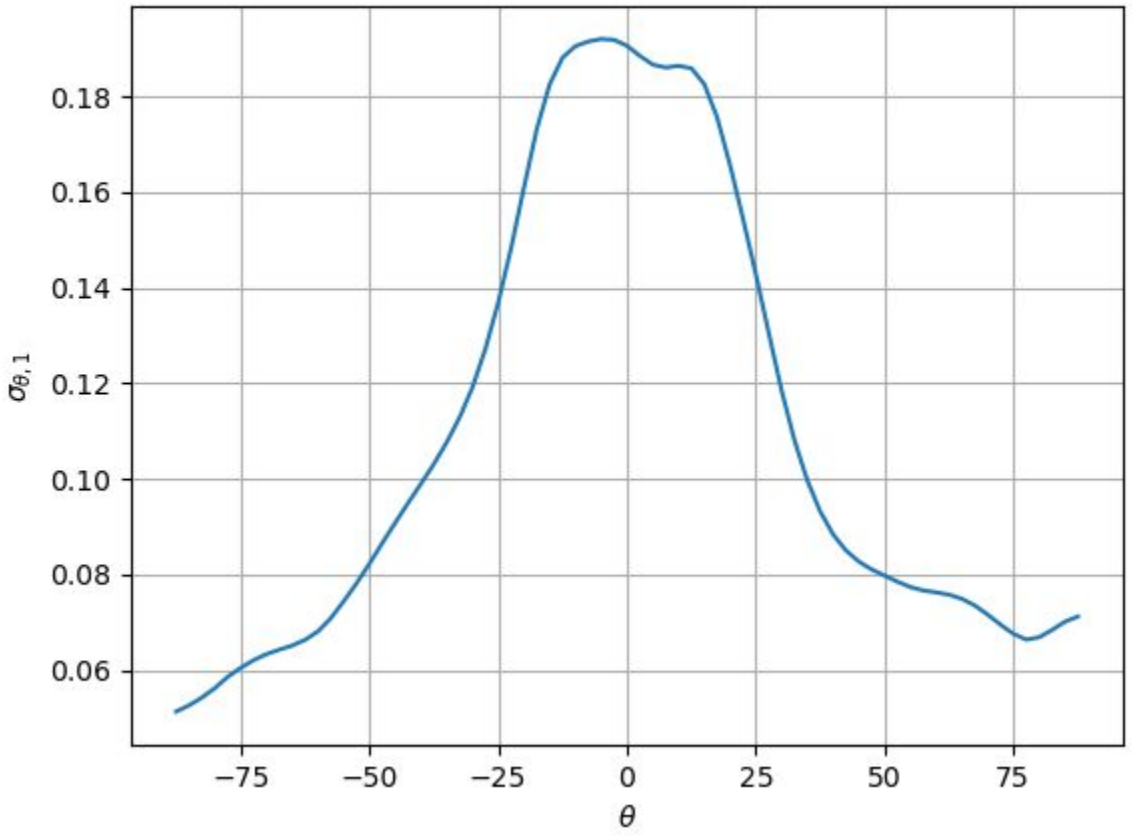


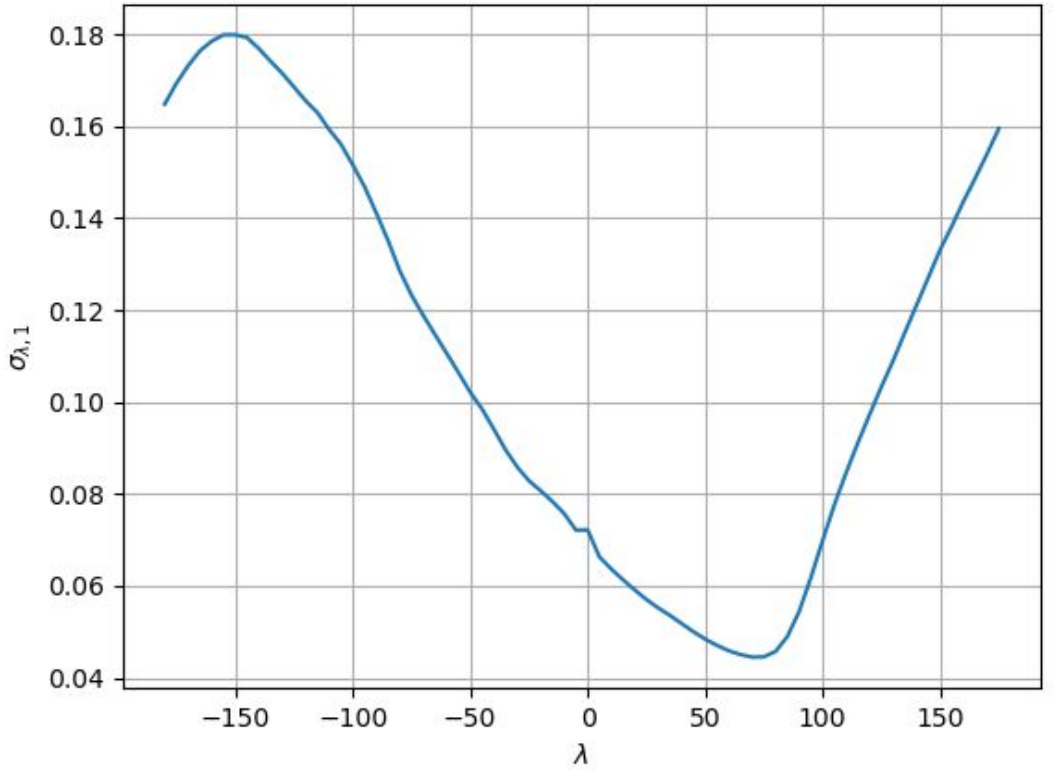




Spatiotemporal Characterization. HOSVD

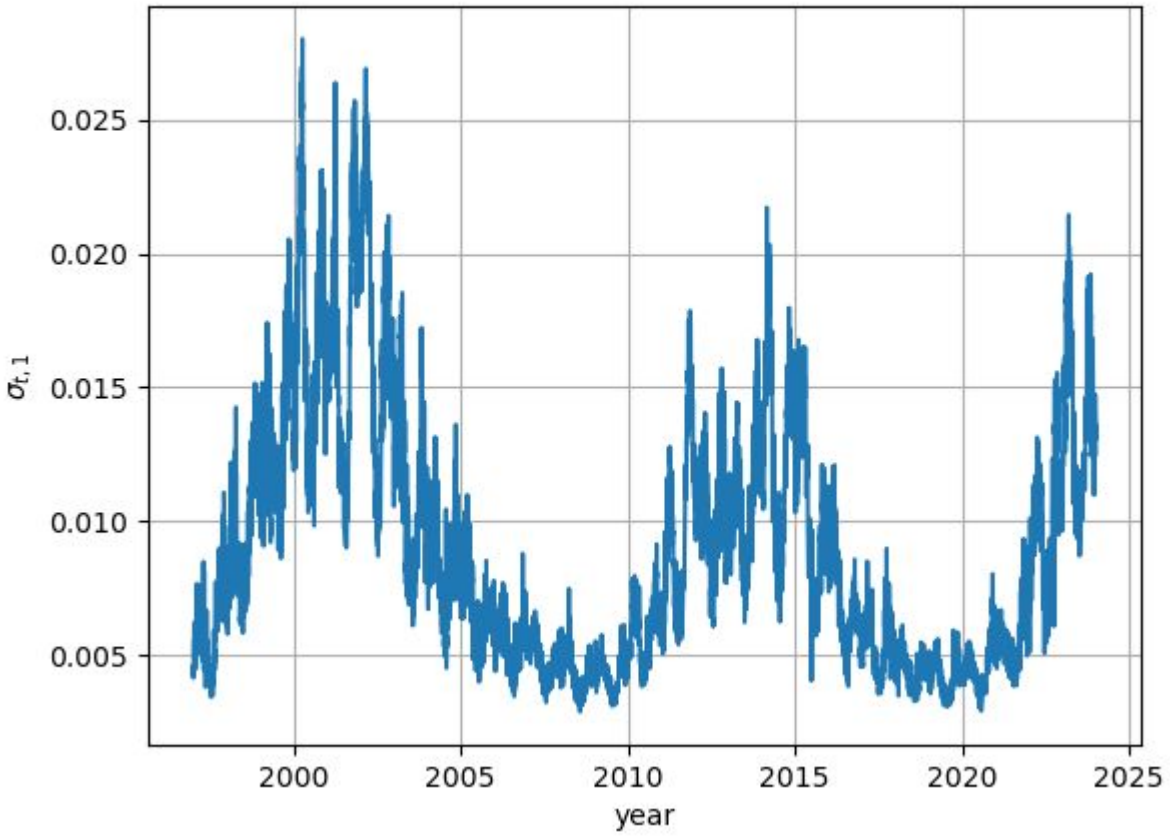
Higher Order SVD. First-mode vectors lat and lon





Spatiotemporal Characterization. HOSVD

Higher Order SVD. First-mode vectors in time



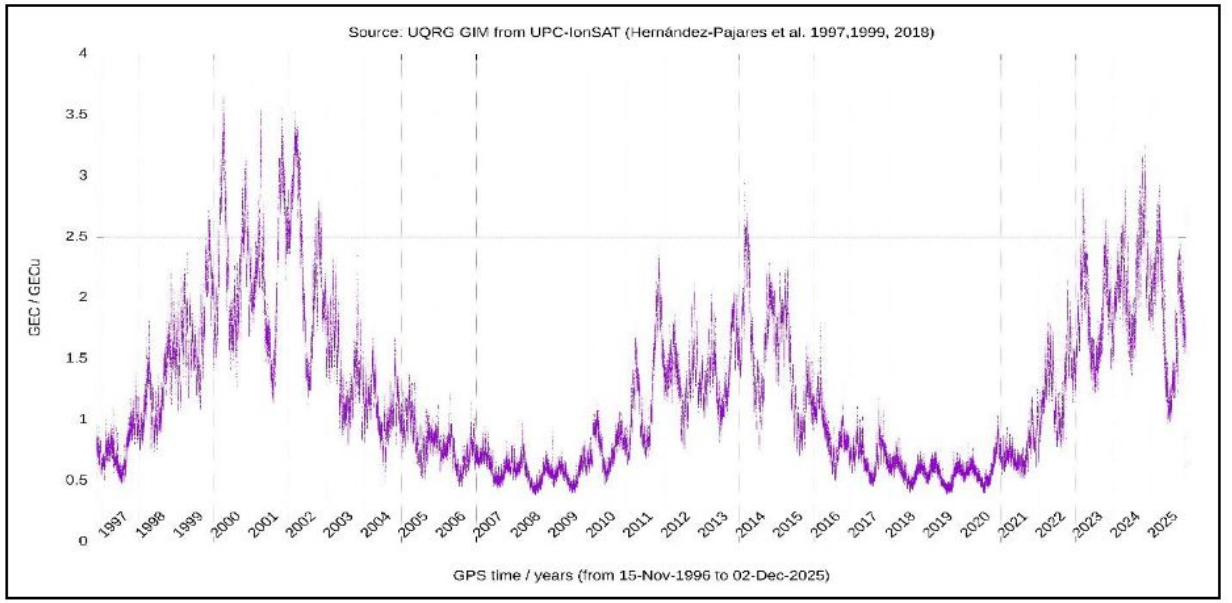
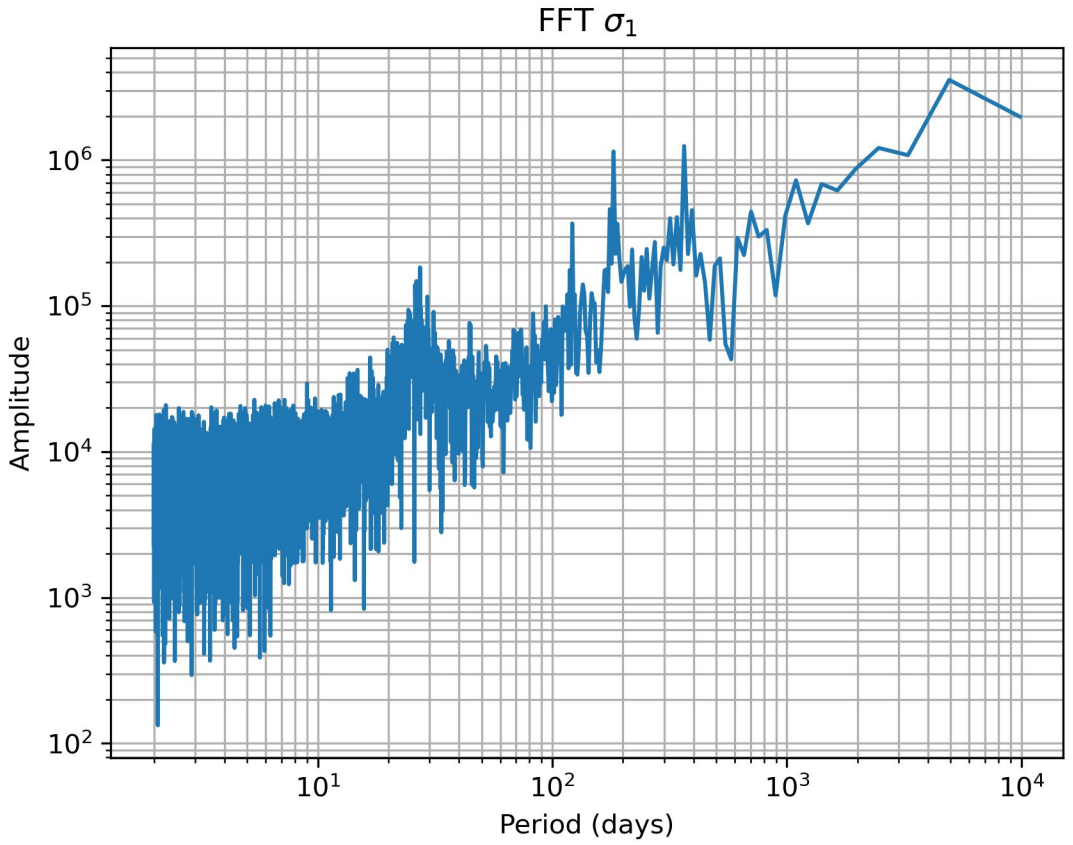


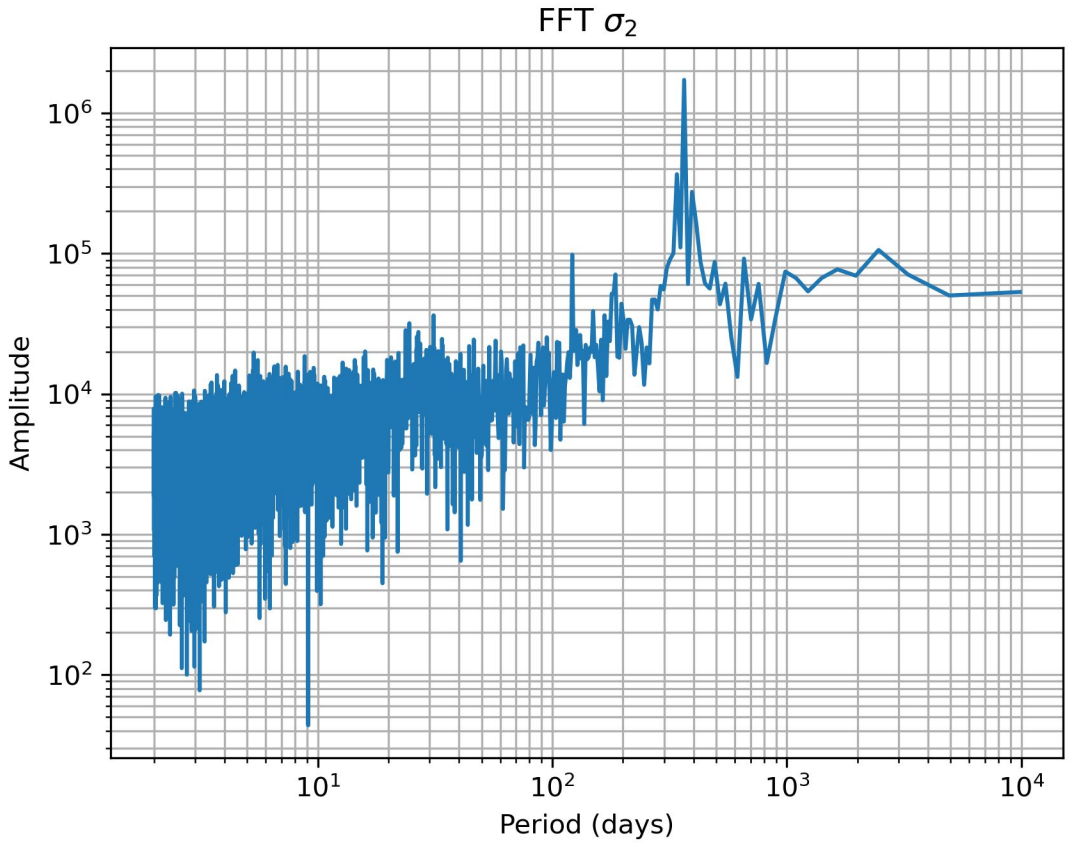
Figure 6 "GEC time series derived, every 15min, from UQRG GIMs since end of 1996"

Spatiotemporal Characterization. HOSVD

Higher Order SVD. VTEC GIMovies

FFT analysis for Time (chrono-vectors).

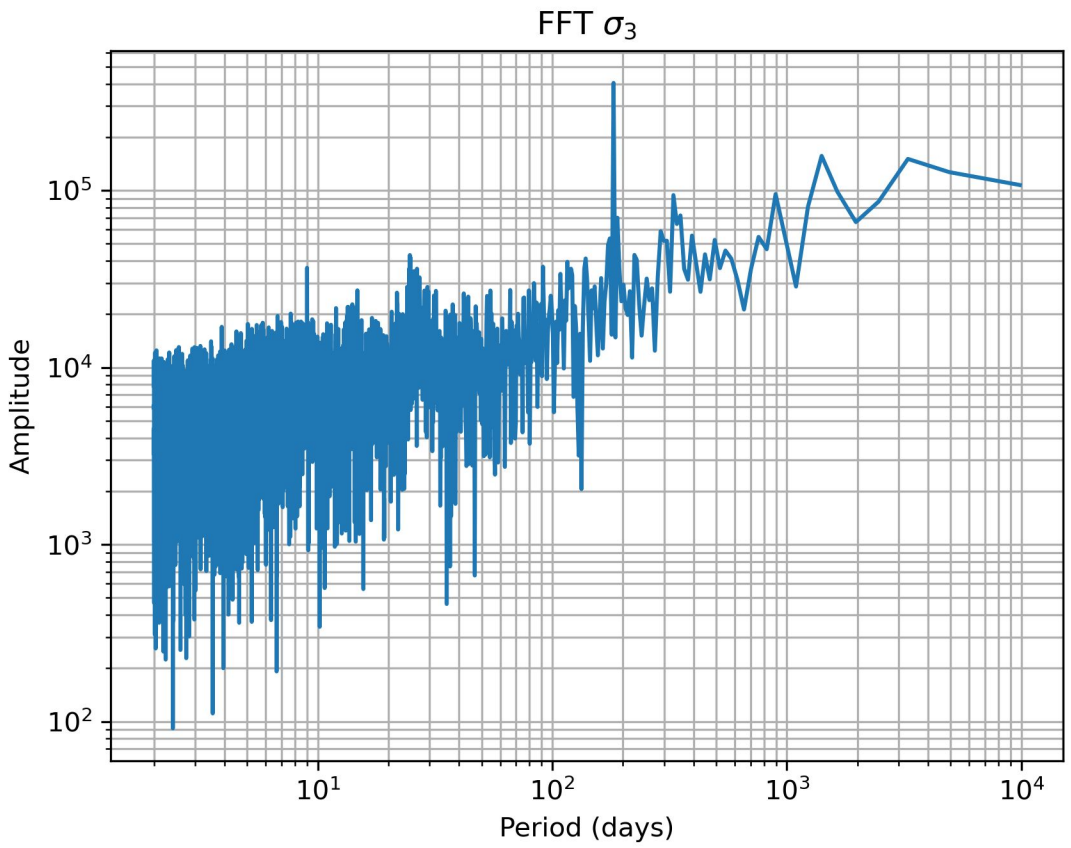


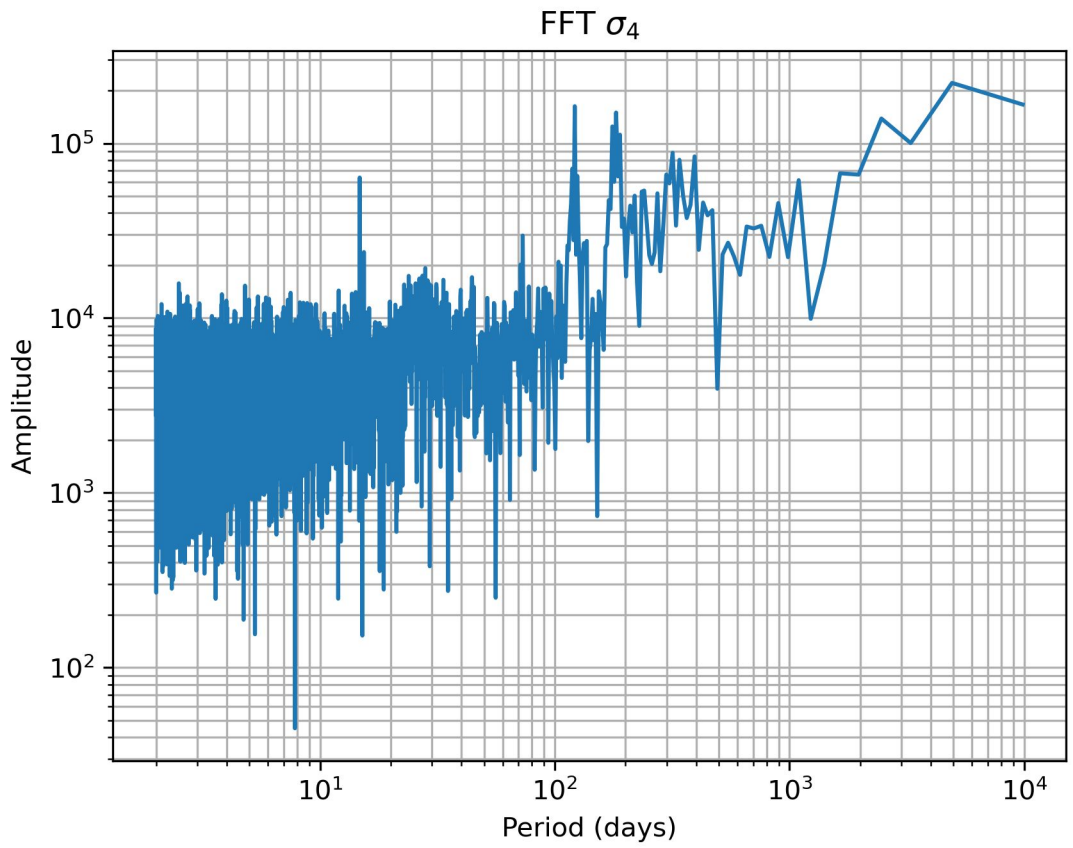


Spatiotemporal Characterization. HOSVD

Higher Order SVD. VTEC GIMovies

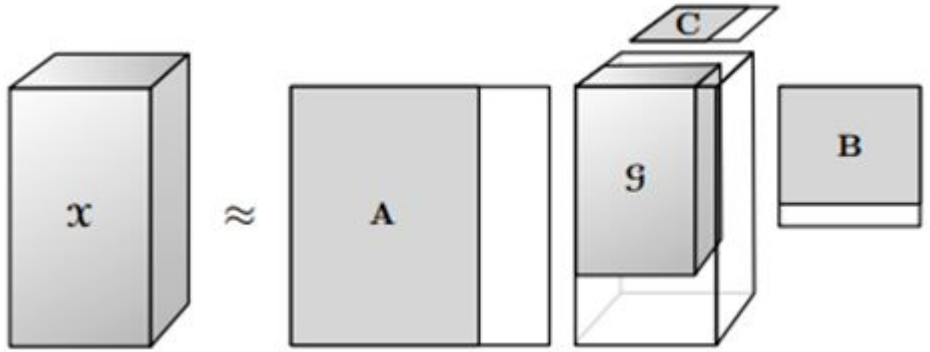
FFT analysis for Time (chrono-vectors).

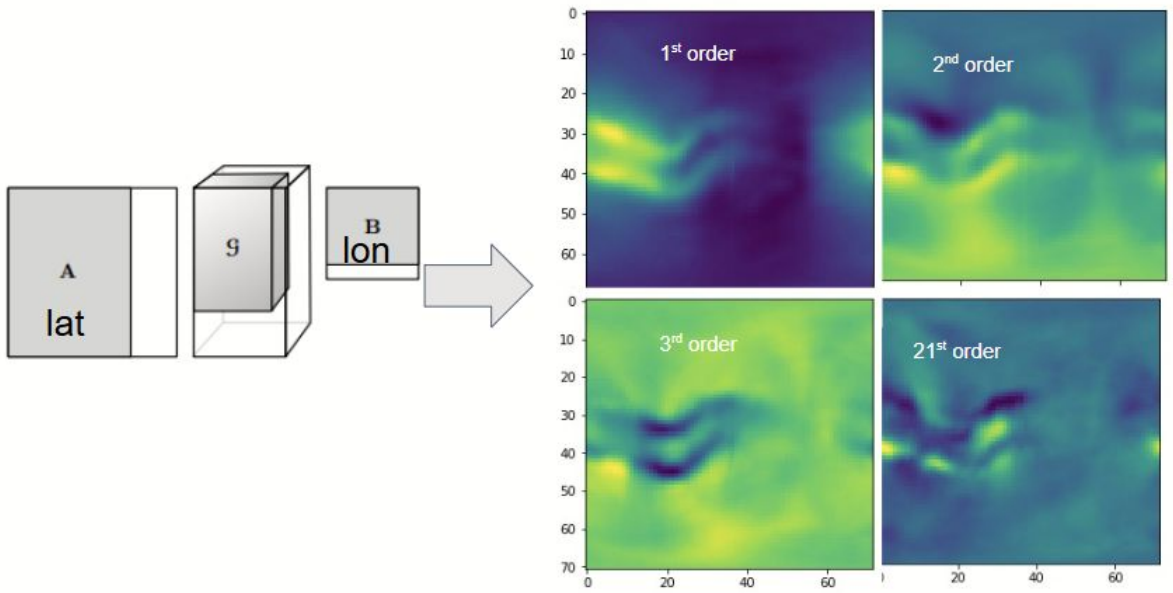




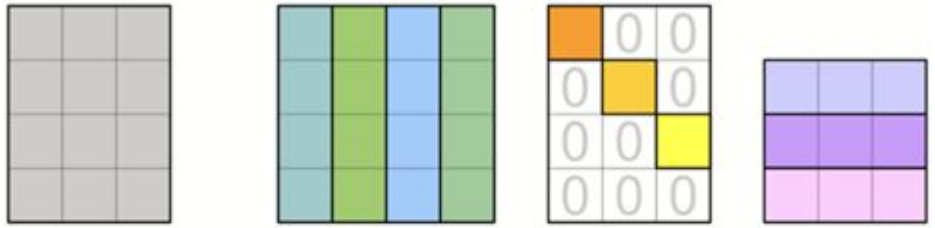
Spatiotemporal Characterization. HOSVD

Higher Order SVD of GIMovies



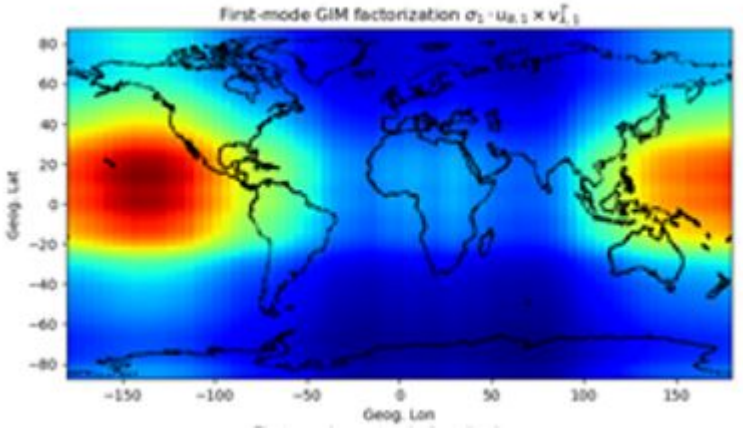


SVD of GIMs



$$\underset{m\times n}{\mathbf{M}} = \underset{m\times m}{\mathbf{U}}\ \underset{m\times n}{\mathbf{\Sigma}}\ \underset{n\times n}{\mathbf{V}^*}$$

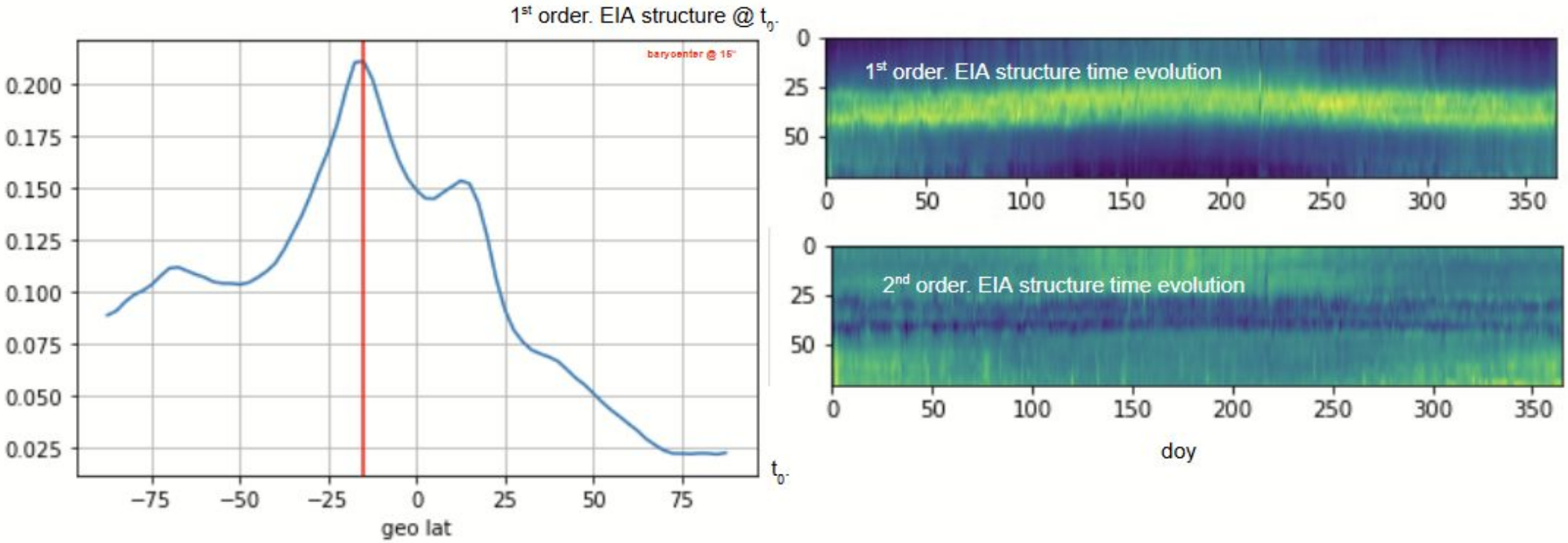
$$M_{1^{st}} = \sigma_1(\vec{u}_1 \otimes \vec{v}_1^T)$$



Spatiotemporal Characterization. HOSVD

Higher Order SVD of GIMovies

The combination (with the outter product) of the latitude and time bases provides images with time-evolution of patterns.

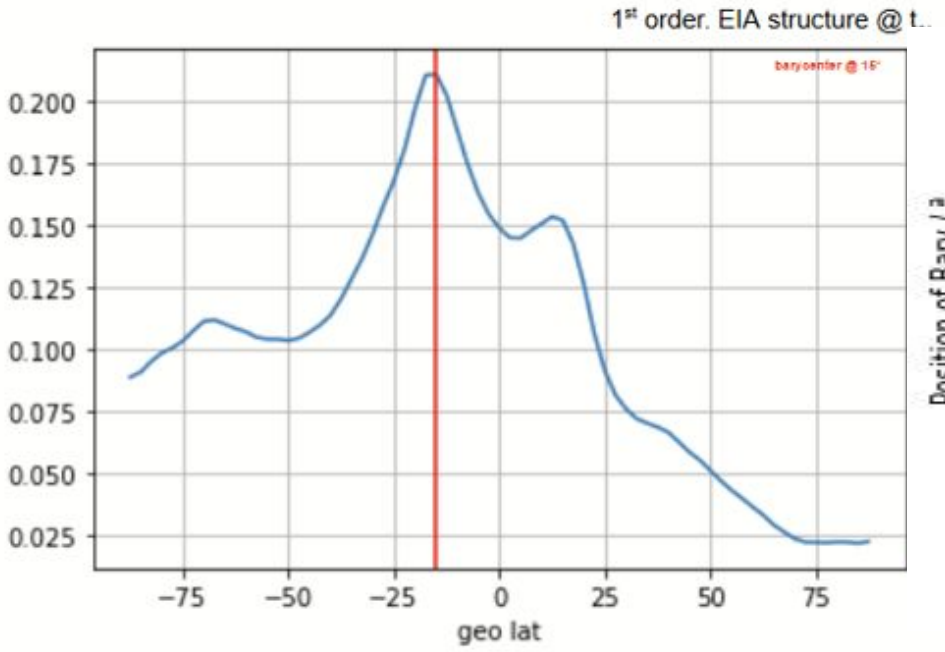


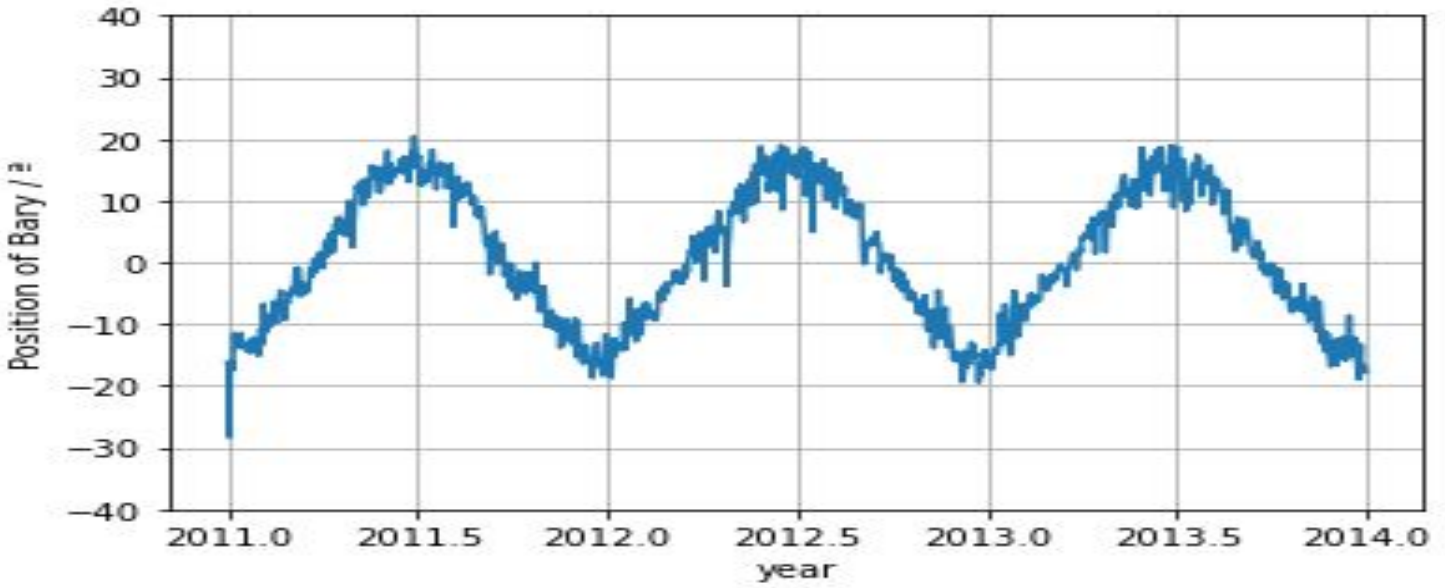
Spatiotemporal Characterization. HOSVD

Higher Order SVD of GIMovies

For example, the first two orders in latitude and time yield the main-mode time evolution of the peak of the EIA:

- Oscillations of the peak indicates the angle witth the ecliptic plane and seasonal patterns.





Spatiotemporal Characterization. HOSVD

Higher Order SVD of GIMovies

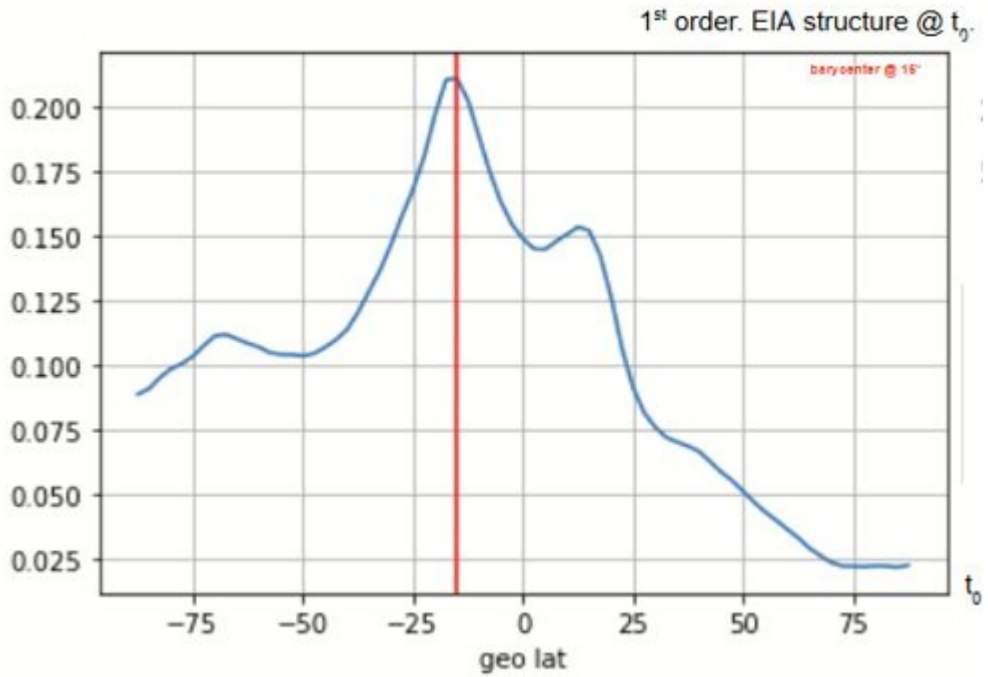
Period	GIM	Potential Source
4000 days (about 11 years)	VTEC, VTEC gradients, Iscale	Solar Cycle
Biannual – 700 days	VTEC, VTEC gradients	(potentially) Quasi-Biennial Oscillation (QBO)
Annual – 365 days	VTEC, VTEC gradients, Iscale	Earth translation around the Sun
Semiannual - 187 days	VTEC, VTEC gradients, Iscale	Earth translation – Earth Axis inclination
Seasonal - 90 days	VTEC, VTEC gradients, Iscale	Earth translation – Seasons
Semimonthly – 14 days	VTEC, VTEC gradients, Iscale	Lunar Tide modulation

Spatiotemporal Characterization. HOSVD

Higher Order SVD of GIMovies

FFT analysis for tropo-vectors □ H4.1 on Thursday

- Is there any geographic scale/s that characterizes the ionosphere?
- What is the scale (if any) with the highest relative weight?



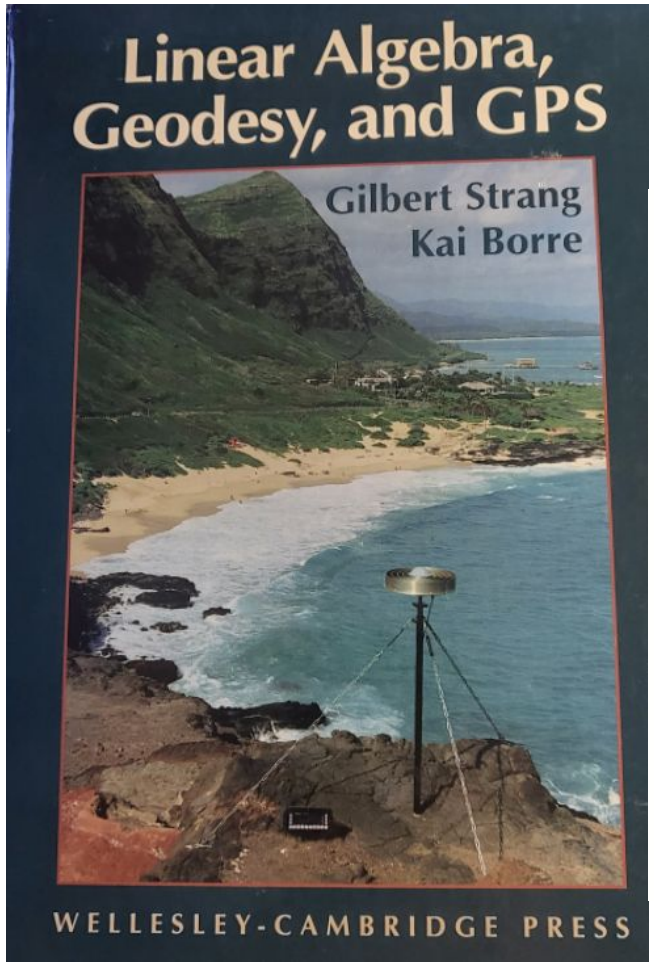
Spatiotemporal Characterization. Conclusions

- SVD for spatial patterns for a given epoch→ Useful for Space Weather, e.g. characterizing abrupt changes in the ionosphere.

- HOSVD→ Long-term patterns, i.e. climatology. Not good for Space Weather events.

Spatiotemporal Characterization. References

A Good introduction to SVD and its applications in Engineering.
Also Strang's Linear Algebra course at MIT on YouTube.



Introduction to HOSVD.

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Tensor Decompositions and Applications*

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Brett W. Bader‡

Abstract. This survey provides an overview of higher-order tensor decompositions, their applications, and available software. A tensor is a multidimensional or N-way array. Decompositions of higher-order tensors (i.e., N-way arrays with $N \geq 3$) have applications in psychometrics, chemometrics, signal processing, numerical linear algebra, computer vision, numerical analysis, data mining, neuroscience, graph analysis, and elsewhere. Two particular tensor decompositions can be considered to be higher-order extensions of the matrix singular value decomposition: CANDECOMP/PARAFAC (CP) decomposes a tensor as a sum of rank-one tensors, and the Tucker decomposition is a higher-order form of principal component analysis. There are many other tensor decompositions, including INDSCAL, PARAFAC2, CANDELINC, DEDICOM, and PARATUCK2 as well as nonnegative variants of all of the above. The N-way Toolbox, Tensor Toolbox, and Multilinear Engine are examples of software packages for working with tensors.

Key words. tensor decompositions, multiway arrays, multilinear algebra, parallel factors (PARAFAC), canonical decomposition (CANDECOMP), higher-order principal components analysis (Tucker), higher-order singular value decomposition (HOSVD)

AMS subject classifications. 15A69, 65F99

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HOSVD and Ionosphere.

Home › Earth, Planets and Space › Article

Analysis of temporal–longitudinal–latitudinal characteristics in the global ionosphere based on tensor rank-1 decomposition

Full paper | Open access | Published: 14 March 2018
Volume 70, article number 39 (2018) Cite this article

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Thank you for your attention!

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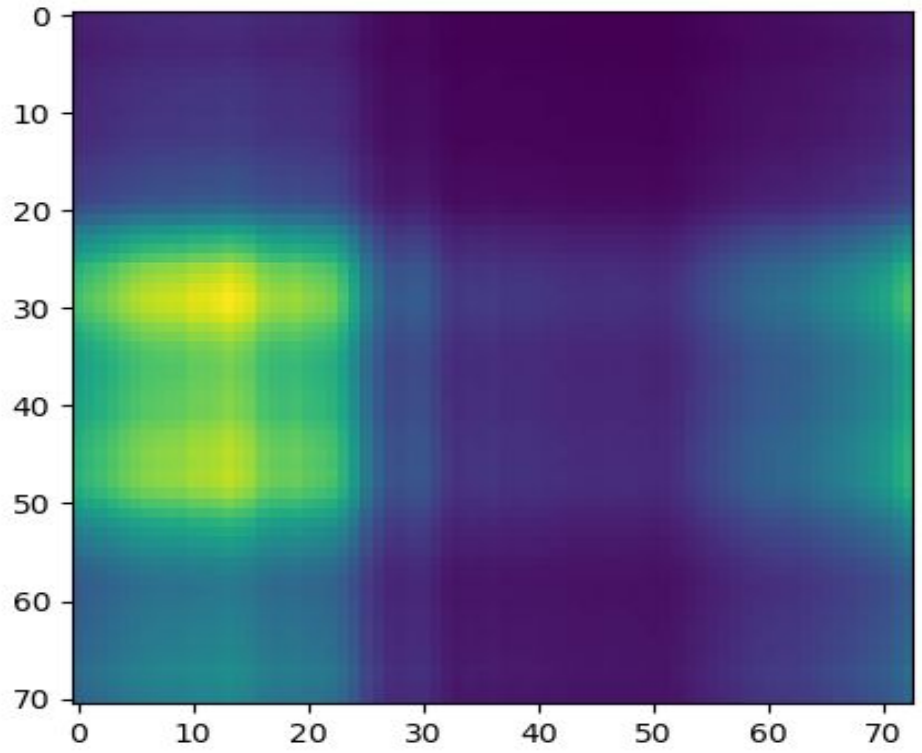
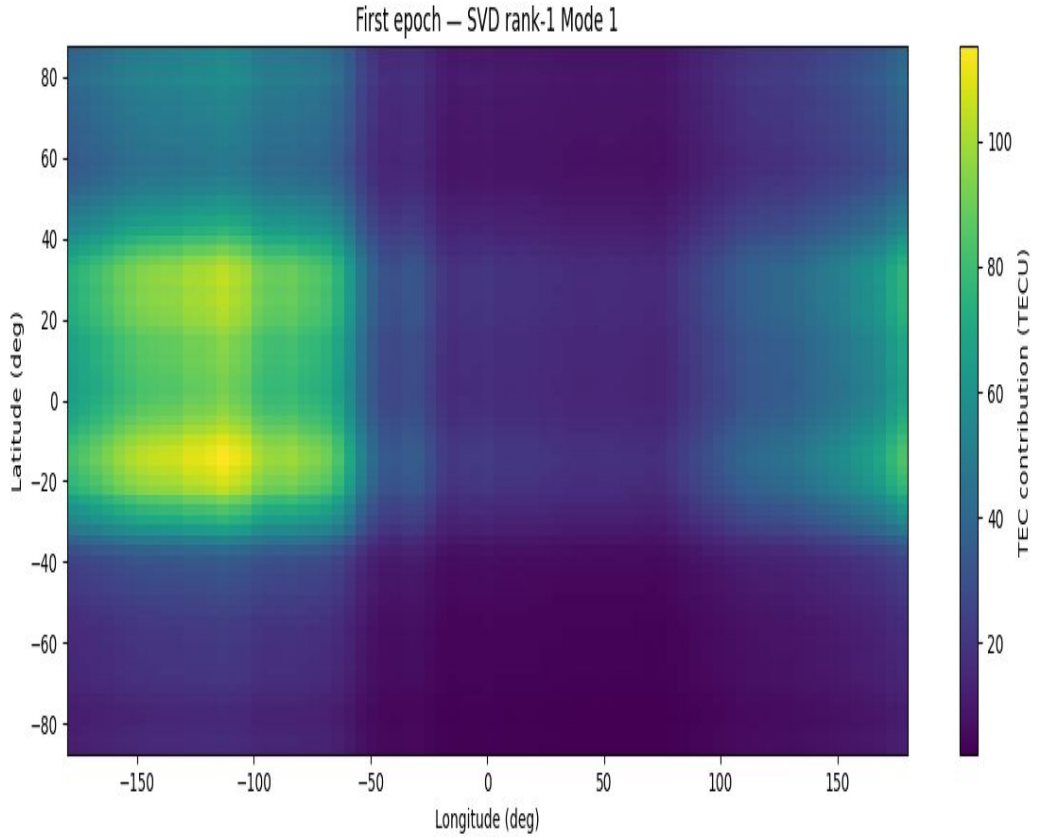
Appendix. NI vs AI

1st mode from DISPEC python script



1st mode from chatgpt

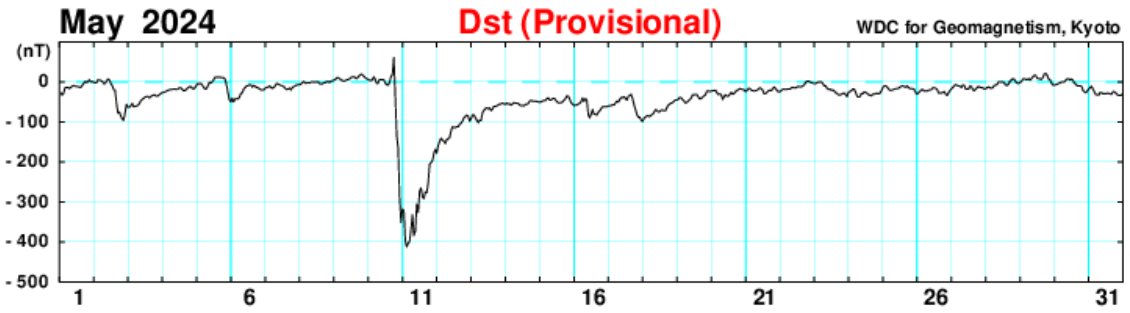


Appendix.

Ionospheric impact on Positioning and navigation

A G5 Geomagnetic Storm occurred in May 11th and 12th 2024.

https://www.spaceweather.gov/noaa-scales-explanation



Geomagnetic Storms

Scale	Description	Effect	Physical measure	Average Frequency (1 cycle = 11 years)
G 5	Extreme	**Power systems:** Widespread voltage control problems and protective system problems can occur, some grid systems may experience complete collapse or blackouts. Transformers may experience damage. **Spacecraft operations:** May experience extensive surface charging, problems with orientation, uplink/downlink and tracking satellites. **Other systems:** Pipeline currents can reach hundreds of amps, HF (high frequency) radio propagation may be impossible in many areas for one to two days, satellite navigation may be degraded for days, low-frequency radio navigation can be out for hours, and aurora has been seen as low as Florida and southern Texas (typically 40° geomagnetic lat.).	Kp = 9	4 per cycle (4 days per cycle)
G 4	Severe	**Power systems:** Possible widespread voltage control problems and some protective systems will mistakenly trip out key assets from the grid. **Spacecraft operations:** May experience surface charging and tracking problems, corrections may be needed for orientation problems. **Other systems:** Induced pipeline currents affect preventive measures, HF radio propagation sporadic, satellite navigation degraded for hours, low-frequency radio navigation disrupted, and aurora has been seen as low as Alabama and northern California (typically 45° geomagnetic lat.).	Kp = 8, including a 9-	100 per cycle (60 days per cycle)
G 3	Strong	**Power systems:** Voltage corrections may be required, false alarms triggered on some protection devices. **Spacecraft operations:** Surface charging may occur on satellite components, drag may increase on low-Earth-orbit satellites, and corrections may be needed for orientation problems. **Other systems:** Intermittent satellite navigation and low-frequency radio navigation problems may occur, HF radio may be intermittent, and aurora has been seen as low as Illinois and Oregon (typically 50° geomagnetic lat.).	Kp = 7	200 per cycle (130 days per cycle)
G 2	Moderate	**Power systems:** High-latitude power systems may experience voltage alarms, long-duration storms may cause transformer damage. **Spacecraft operations:** Corrective actions to orientation may be required by ground control; possible changes in drag affect orbit predictions. **Other systems:** HF radio propagation can fade at higher latitudes, and aurora has been seen as low as New York and Idaho (typically 55° geomagnetic lat.).	Kp = 6	600 per cycle (360 days per cycle)
G 1	Minor	**Power systems:** Weak power grid fluctuations can occur. **Spacecraft operations:** Minor impact on satellite operations possible. **Other systems:** Migratory animals are affected at this and higher levels; aurora is commonly visible at high latitudes (northern Michigan and Maine).	Kp = 5	1700 per cycle (900 days per cycle)

Appendix.
Ionospheric impact on Positioning and navigation

A G5 Geomagnetic Storm occurred in May 11th and 12th 2024.
VTEC, VTEC gradient RMS and height error (single-frequency PPP).

