DISPEC - Summer School – September 2026

Ionospheric Propagation

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Lecture motivation:

Why do we model propagation?

Radio signals traversing the ionosphere are modified by refraction, delay, absorption, polarization changes and scattering.

These effects influence GNSS positioning, satellite communications, HF radio, over-the-horizon radar and remote sensing.

Accurate propagation modelling is therefore essential for predicting system performance and interpreting ionospheric observations**.**



Ionospheric Propagation

Lecture content:

- **Ionospheric layer:** a short description
- **Propagation of an EM wave in ionosphere**
- **Link crossing ionosphere layer:** propagation effects
  - Refraction
  - Attenuation
  - Effect on polarisation
  - Scintillation
- **Link by ionospheric reflection**

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Ionospheric layer: a short description

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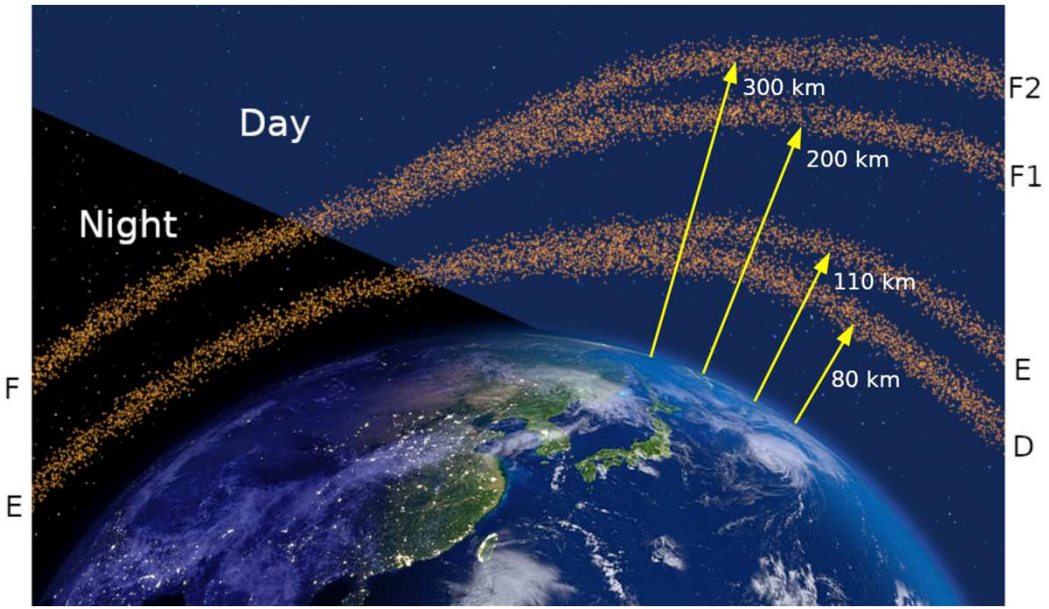
Short introduction: IONOSPHERE

In 1902, Kennely and Heaviside discovered ionised layers by reflection of radiowaves.

Since 1922, different layers have been defined (D, E, F ...).



Layer	Height (km)	Density (e/m^3)
D	60 - 90	10^8 to 10^{10}
E	90 - 120	10^{10} to 10^{11}
F (night)	F1 : 120 - 250	10^{11} to 10^{12}
F1 & F2 (day)	F2 : 250 - 500	

In reality, the different layers are not separated, the variation of ionisation is continuous.

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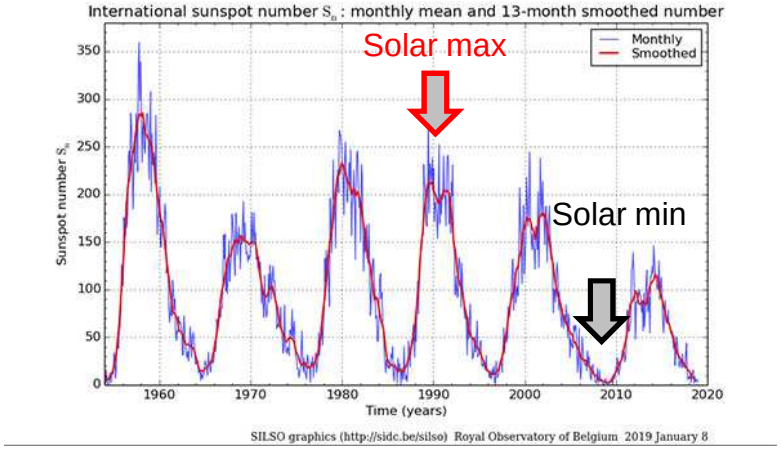
Short introduction: IONOSPHERE



Vertical ionosphere profile: variable with regard to

- night/day
- season/ solar activity
- latitude

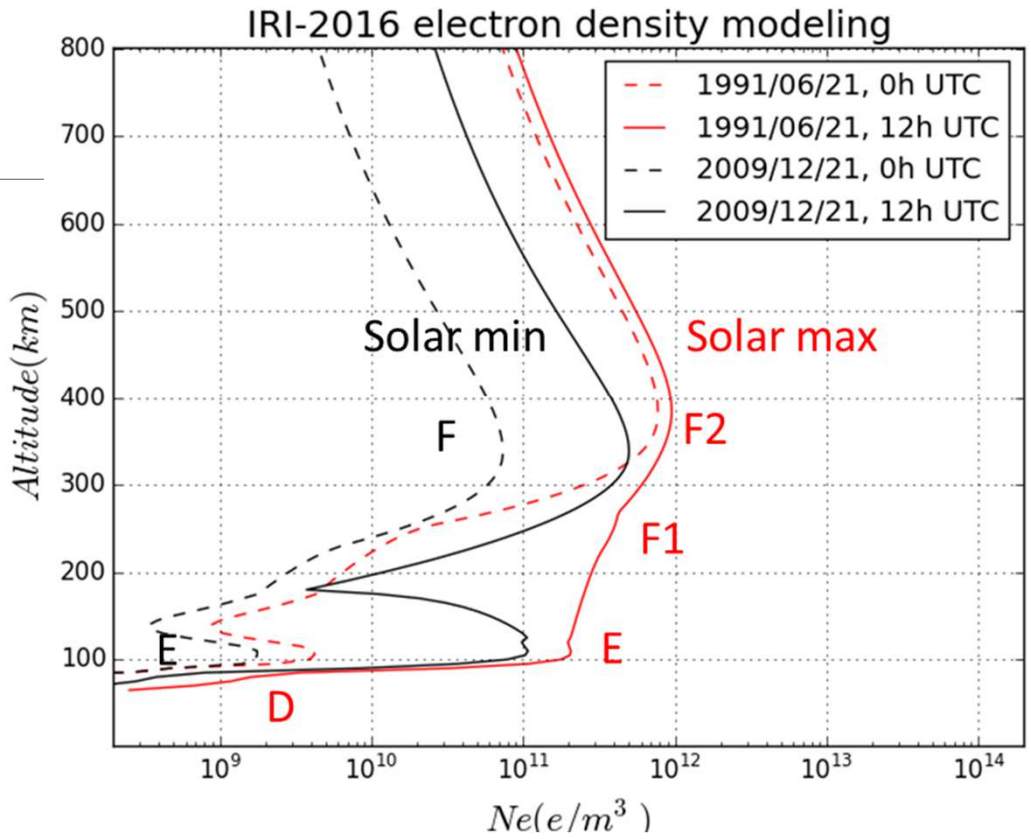
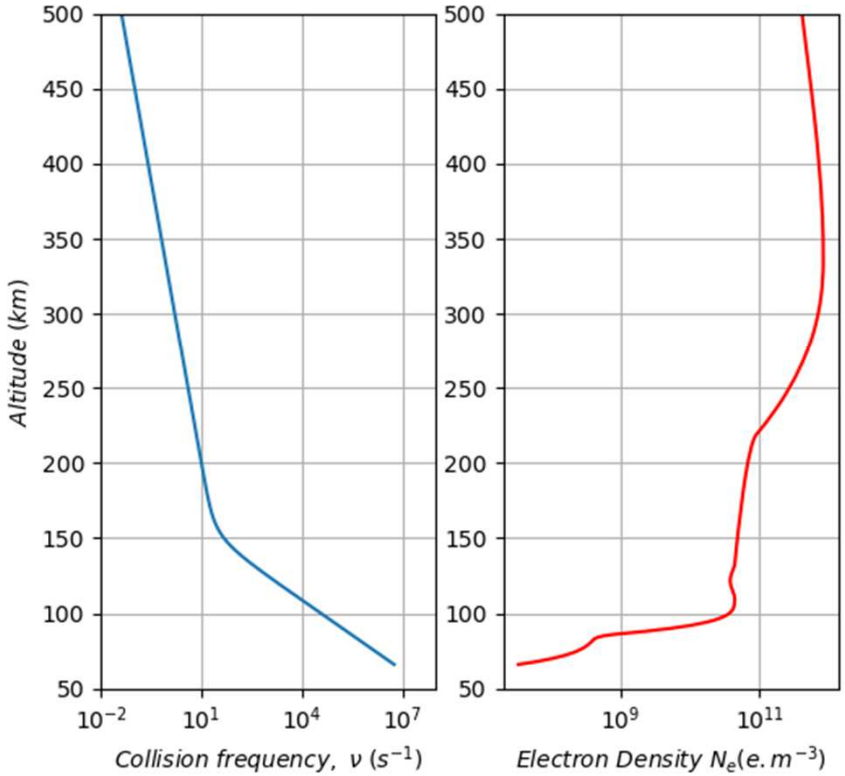


Figure : ionisation profile for **mean** latitudes

Curves from IRI 2016 modeling



Introduction: IONOSPHERE



Curves from IRI 2016 modeling

Figure :

Left – typical, collision frequency profile

Right – typical ionisation profile for mean latitudes

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Propagation of an EM wave in ionosphere

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Propagation of an EM wave in ionosphere

Equations without Earth magnetic field influence:

- Maxwell equation

$$\vec{\nabla} \times \vec{H} = \vec{J} + \varepsilon_0 \frac{d\vec{E}}{dt}$$

- Movement equation of electrons

$$m\frac{d\vec{V}}{dt} + mv\vec{V} = q\vec{E}$$

$$with \quad \vec{J} = \rho\vec{V} = N_e q\vec{V}$$

Current induced by electrons movement

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Propagation of an EM wave in ionosphere

Equations without Earth magnetic field influence:

•Movement equation of electrons

$$m\frac{d\vec{V}}{dt} + m\nu\vec{V} = q\vec{E}$$

force (average rate of loss)

force due to electron motion

electric force

with

$\vec{V}$: particule speed

m : particule mass

$\vec{E}$: electric field

ν : electron-neutral collision frequency

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Propagation of an EM wave in ionosphere

Equations without Earth magnetic field influence:

$\vec{V}$: particule speed
m : particule mass
$\vec{E}$: propagated electric field
$\vec{H}$: propagated magnetic field
ρ : volume charge
v : electron-neutral collision frequency
N_e : particules number per m^3
q : particule charge
$\vec{J}$: induced currents in ionized gaz

$$m\frac{d\vec{V}}{dt} + mv\vec{V} = q\vec{E} \quad (1)$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \varepsilon_0 \frac{d\vec{E}}{dt} \quad (2)$$

$$with \quad \vec{J} = \rho\vec{V} = N_e q\vec{V}$$

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Propagation of an EM wave in ionosphere

Harmonic resolution : $\boxed{\frac{d}{dt} \rightarrow i\omega}$

(1) $m\frac{d\vec{V}}{dt} + mv\vec{V} = q\vec{E} \longrightarrow (i\omega m + mv)\vec{V} = q\vec{E}$

i.e. $\vec{V} = \frac{q}{(i\omega m + mv)}\vec{E}$

(2) $\vec{\nabla} \times \vec{H} = \rho\vec{V} + \varepsilon_0\frac{d\vec{E}}{dt} \longrightarrow \vec{\nabla} \times \vec{H} = N_e q\vec{V} + i\omega\varepsilon_0\vec{E}$

$\Rightarrow \vec{\nabla} \times \vec{H} = \left(N_e q\frac{q}{(i\omega m + mv)} + i\omega\varepsilon_0\right)\vec{E}$

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Propagation of an EM wave in ionosphere

$$\vec{\nabla} \times \vec{H} = \left(N_e q \frac{q}{(i\omega m + mv)} + i\omega\varepsilon_0 \right) \vec{E} = i\omega \varepsilon_r' \varepsilon_0 \vec{E}$$

Complex relative permittivity

$$= \left(N_e \frac{q^2}{i\omega\varepsilon_0 m(i\omega + v)} + 1 \right) i\omega\varepsilon_0 \vec{E} = \left(N_e \frac{q^2(v - i\omega)}{i\omega\varepsilon_0 m(\omega^2 + v^2)} + 1 \right) i\omega\varepsilon_0 \vec{E}$$

$$= \left[\left(1 - \frac{N_e q^2}{\varepsilon_0 m(\omega^2 + v^2)} \right) - i \frac{N_e q^2 v}{\varepsilon_0 m\omega(\omega^2 + v^2)} \right] i\omega\varepsilon_0 \vec{E}$$

Phase index without Earth magnetic field influence:

$$\varepsilon_r' = n_\varphi^2 = \left[\left(1 - \frac{N_e q^2}{\varepsilon_0 m(\omega^2 + v^2)} \right) - i \frac{N_e q^2 v}{\varepsilon_0 m\omega(\omega^2 + v^2)} \right]$$

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Propagation of an EM wave in ionosphere

Critical pulsation of plasma:

Choosing: $$\omega_c^2 = \frac{N_e q^2}{m\epsilon_0}$$

then: $$\varepsilon_r' = n_\varphi^2 = \left(1 - \frac{\omega_c^2}{(\omega^2 + v^2)}\right) - i\frac{\omega_c^2 v}{\omega(\omega^2 + v^2)}$$

In ionosphere, as first approximation : $\nu \approx 0$ $$\varepsilon_r' = n_\varphi^2 \simeq \left(1 - \frac{\omega_c^2}{\omega^2}\right)$$

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Propagation of an EM wave in ionosphere

Maxwell equations:

$$\vec{\nabla} \times \vec{E} = -i\omega\mu_0 \vec{H}$$
$$\vec{\nabla} \times \vec{H} = i\omega n_\varphi^2 \epsilon_0 \vec{E}$$

⟶ Dispersion equation :

$$k^2 = \frac{\omega^2}{c^2} n_\varphi^2 = k_0^2 n_\varphi^2$$

$$k^2 \simeq k_0^2 \left(1 - \frac{\omega_c^2}{\omega^2}\right) \quad \textit{(Klein-Gordon relation)}$$

Propagation of a Plane Wave : $\overrightarrow{E}(r,t) = \overrightarrow{E_0} e^{i(\omega t - \vec{k}.\vec{r})} = \overrightarrow{E_0} e^{i(\omega t - n_\varphi \overrightarrow{k_0}.\vec{r})}$

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Propagation of an EM wave in ionosphere

Critical pulsation of plasma is related to the layer of ionosphere:

for $f < f_c$, (or $\omega < \omega_c$) wave propagation in ionosphere is impossible because

$n_\varphi = \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}$ is imaginary:

the propagated wave is evanescent and decay exponentially with distance.

Considering constants:

$$\left.\begin{aligned} q = -e = 1{,}6.10^{-19}\,C \\ m = 0{,}91.10^{-30}\,kg \end{aligned}\right\} \Rightarrow$$

$$f_c = \frac{\omega_c}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{N_e e^2}{m\epsilon_0}} \Rightarrow f_c \simeq 9\sqrt{N_e}$$

Note : ω_c is also called Langmuir frequency

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Link crossing ionosphere layer

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Link crossing ionosphere layer

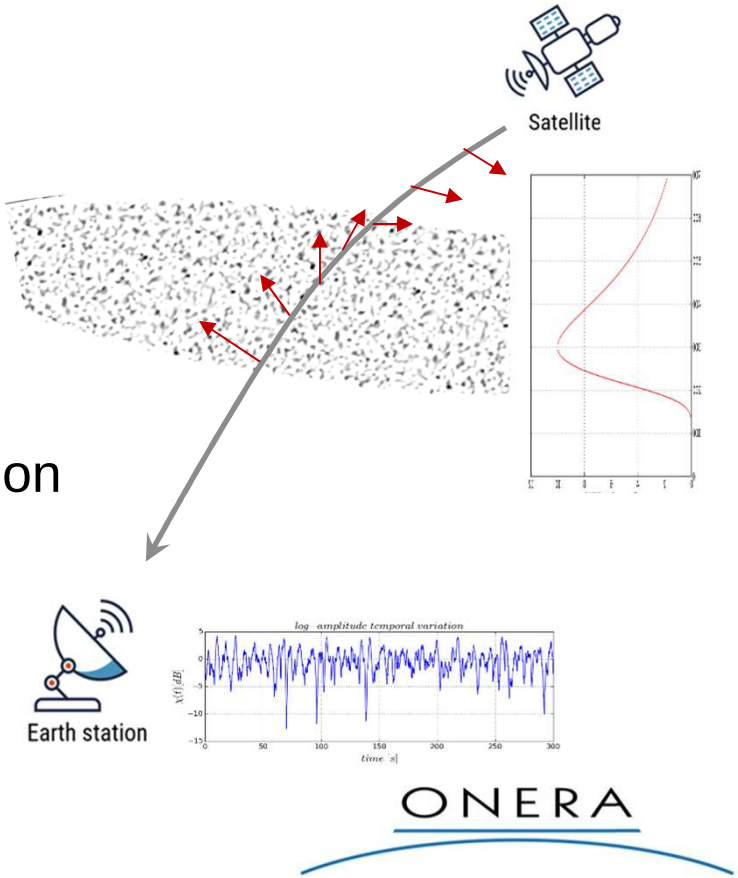
Ionosphere Effects on Earth-satellite links

1. Refraction
– Path delay
– Modified angle of arrival

2. Absortion/attenuation
– Losses on budjet links

3. Polarisation effect: Faraday rotation
– Losses in polarised reception

4. Scintillation
– Losses on budjet links





Ionosphere effect on propagation: 1. Refraction

Refraction: Ionospheric delay

$$\delta L = \int_{s_{min}}^{s_{max}} \left(n_g(s) - 1\right) ds$$

Ionosphere is a dispersive medium, group index must be considered !

$$n_g = \frac{1}{n_\varphi} = \left[1 - \left(\frac{f_c}{f}\right)^2\right]^{-1/2} \underset{f \gg f_c}{\approx} 1 + \frac{1}{2}\left(\frac{f_c}{f}\right)^2$$

So with $f_c \simeq 9\sqrt{N_e}$

$$n_g - 1 \approx \frac{1}{2}\left(\frac{f_c}{f}\right)^2 \approx \frac{1}{2}\frac{81\,N_e}{f^2} \approx \frac{40\,N_e}{f^2}$$

And the difference of path length

$$\delta L = \int_{S_{min}}^{S_{max}} \left(n_g(s) - 1\right) ds \approx \frac{40}{f^2} \int_{S_{min}}^{S_{max}} N_e(s) ds$$

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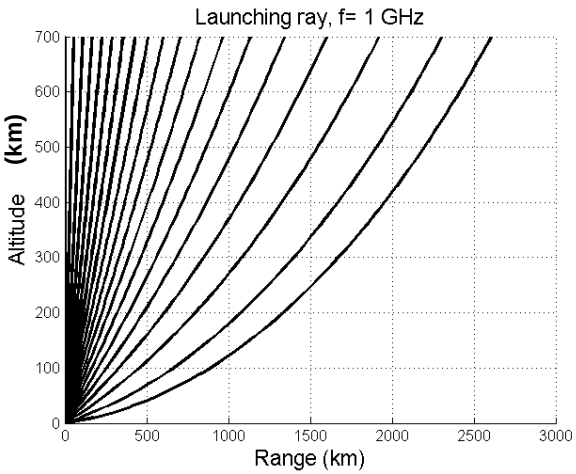
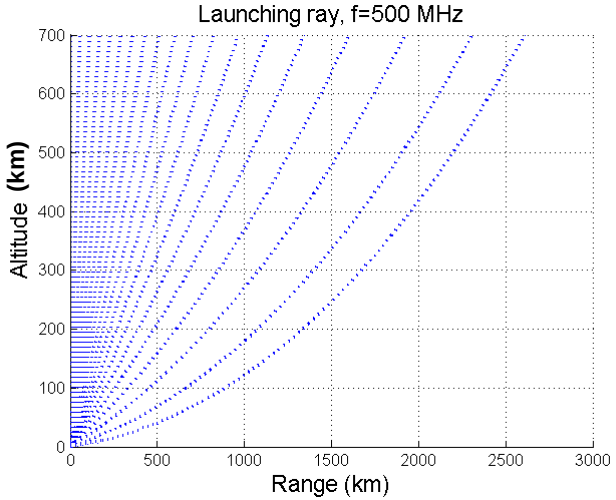
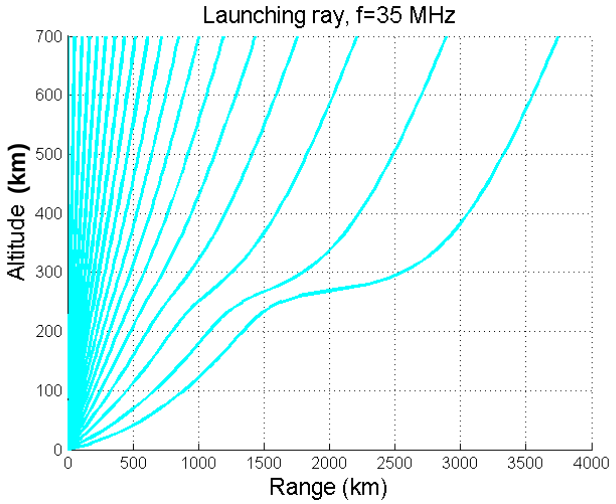
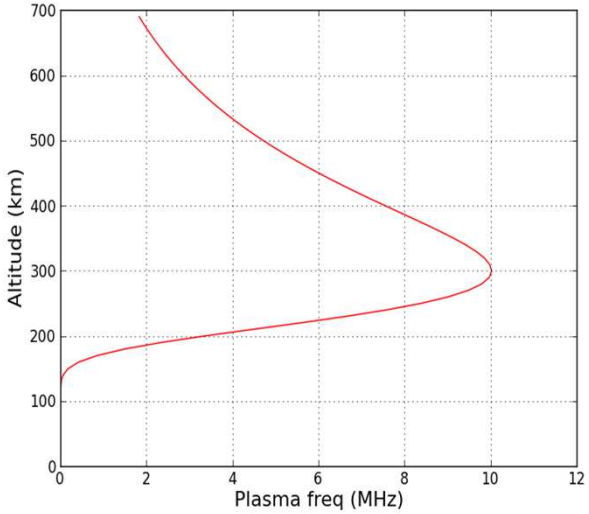
Ionosphere effect on propagation: 1. Refraction

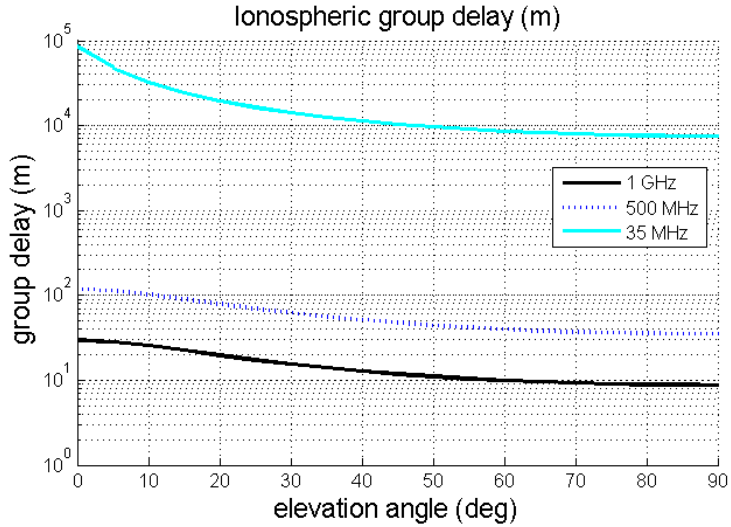






Example of launching ray in canonical ionosphere $(f_c^{max} = 10\ MHz)$





Corresponding ionospheric delay vs elevation

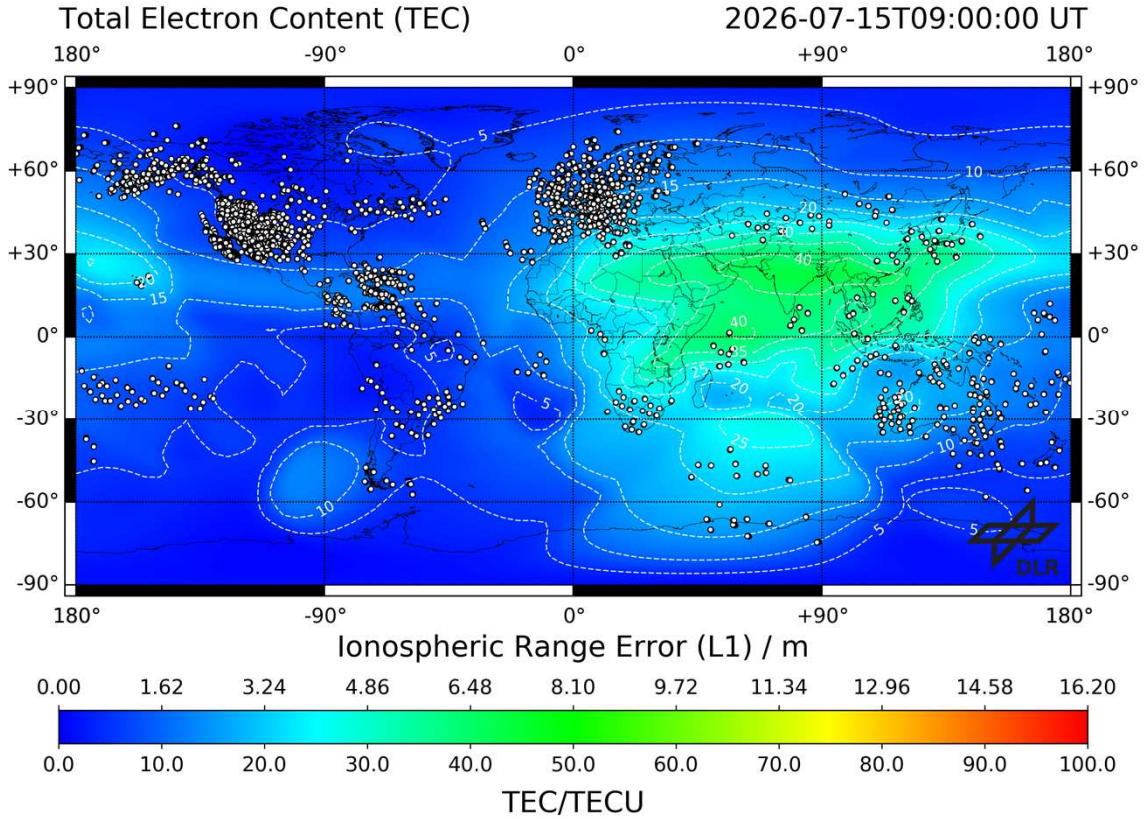


Ionosphere effect on propagation: 1. Refraction

Application to GPS frequency L1(1575.42)

$$\delta L \approx \frac{40}{f^2} STEC$$

16cm or 0.54ns for 1 TECU



From :
https://impc.dlr.de/products/total-electron-content/near-real-time-tec/near-real-time-tec-maps-global



Propagation in ionosphere: 2. Attenuation

From basic equation:

- **Medium index** $\varepsilon_r' = n_\varphi^2 = \left[\left(1 - \frac{N_e q^2}{\varepsilon_0 m(\omega^2 + v^2)}\right) - i\frac{N_e q^2 v}{\varepsilon_0 m\omega(\omega^2 + v^2)}\right]$

- **and propagation** $\overrightarrow{E}(r,t) = \overrightarrow{E_0}\, e^{i(\omega t - n_\varphi \overrightarrow{k_0}.\vec{r})}$

Losses/ attenuation:

due to collisions between electrons and neutral molecules

from imaginary part of index

attenuation with range: $\alpha(dB/km) = 4{,}6.10^{-2}\frac{N_e v}{\omega^2 + v^2}$

For $\omega >> v$, usual relation: $\alpha(dB/km) = 1{,}16.10^{-3}\frac{N_e v}{f^2}$

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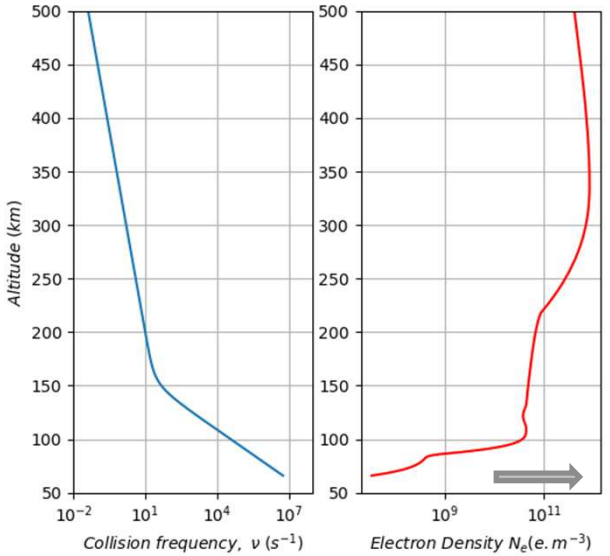
Propagation in ionosphere: 2. Attenuation

Attenuation

$$\alpha(dB/km) = 1{,}16.10^{-3}\frac{N_e v}{f^2}$$

- Problem

product of collision frequency with electron density often unknown



$N_e v$ maximum in D layer

- Coarse estimation :

Considering D layer (between 80 and 100 km)

$N_e v\ integral \approx 1{,}8.10^{18}$

$$A_\varphi(dB) \approx \frac{2100}{f_{MHz}^2\sqrt{sin^2\varphi + 2z/a}}$$

$z \approx 100\ km$; a: Earth radius

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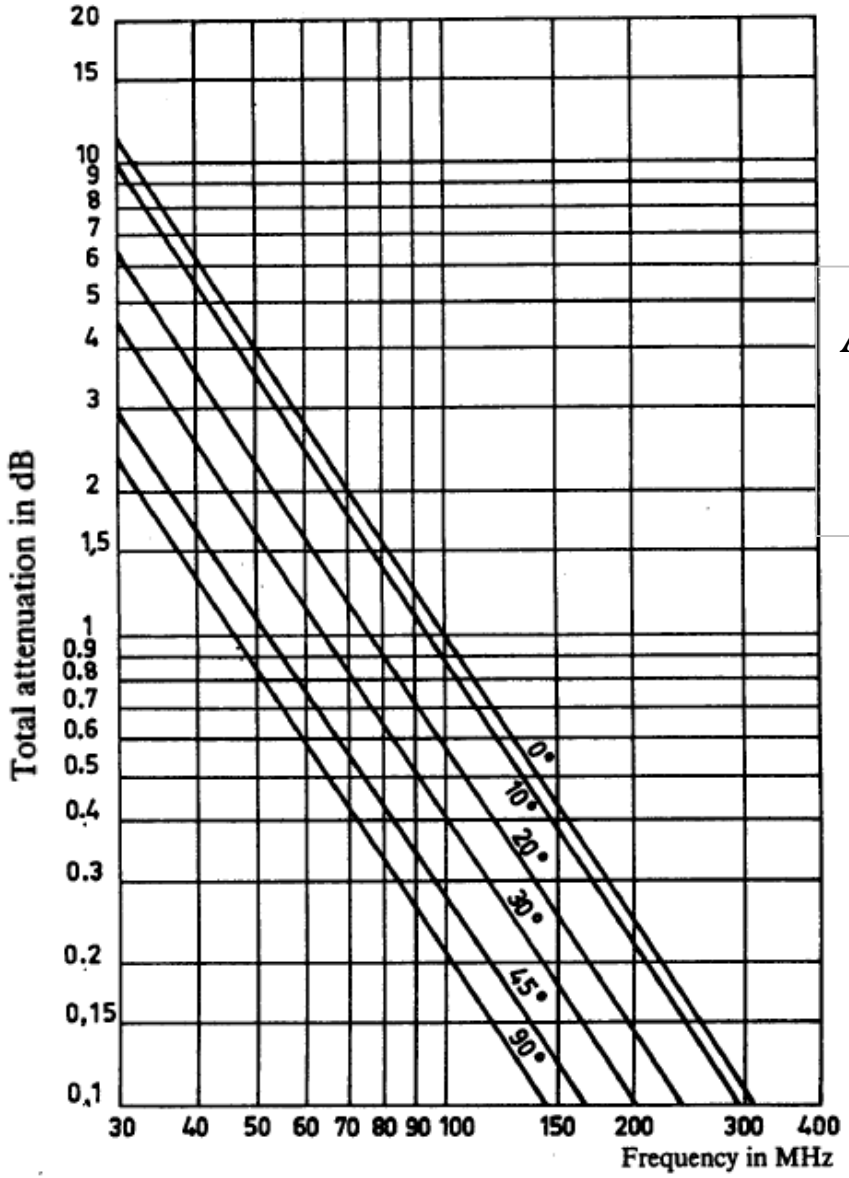
Propagation in ionosphere: 2. Attenuation



During the day

$$A_{\varphi}(dB) \approx \frac{2100}{f_{MHz}^{2}\sqrt{sin^{2}\varphi + 2z/a}}$$

$z \approx 100\ km$; a: Earth radius

During the night, attenuation is about 15 dB less

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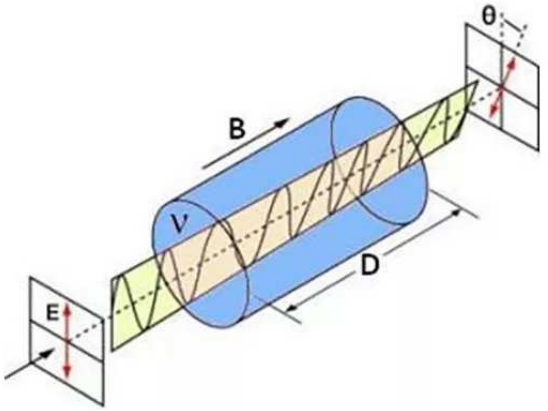
Propagation in ionosphere: 3. Polarization

Faraday rotation

Earth magnetic field creates birefringence

So a rotation of the polarisation plane (Faraday effect)
Approximated rotation angle expression :



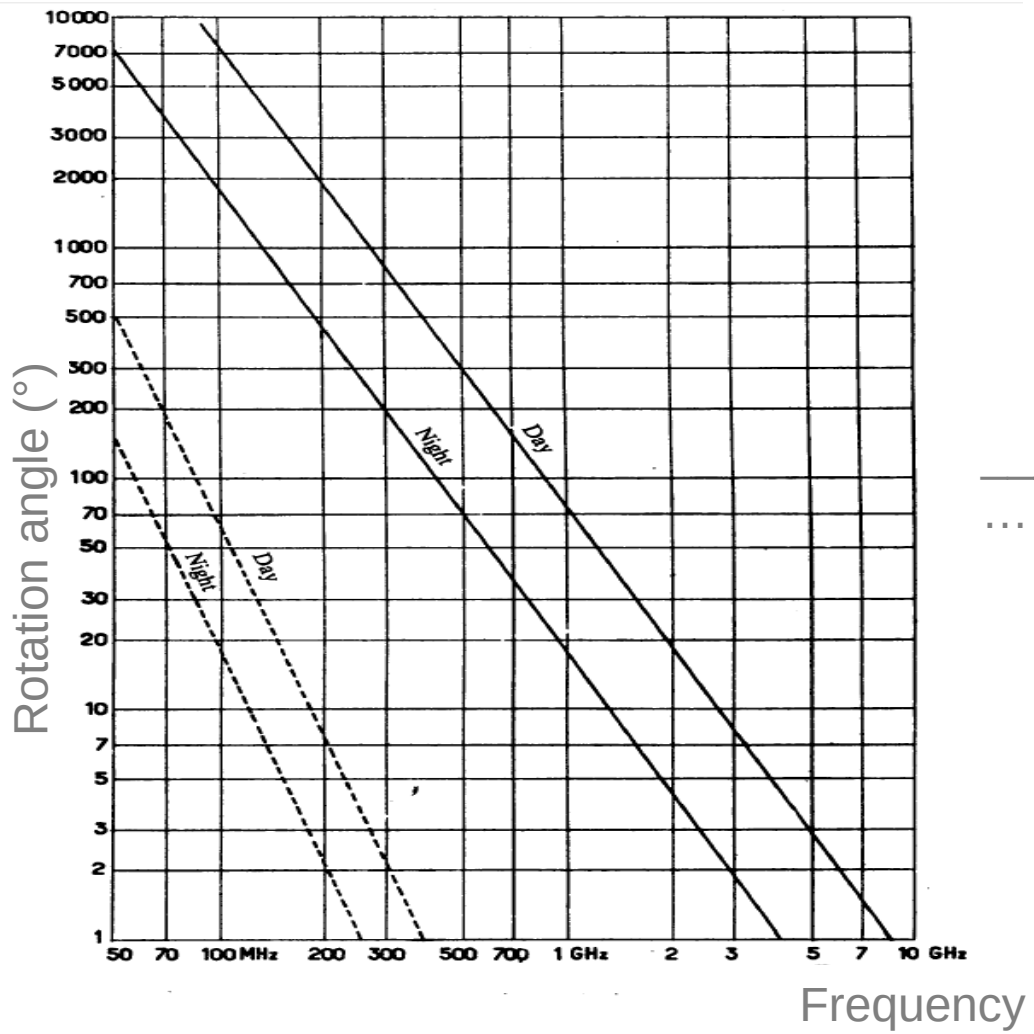
$$\Omega = \frac{K}{f^2}\int_{S_{min}}^{S_{max}} N_e(s)(\vec{B}.\vec{s})ds \quad \Rightarrow \quad \Omega \simeq \frac{K}{f^2} B_{av}\, STEC$$

B_{av} average value of Earth magnetic field component parallel to wave vector.
f : frequency (Hz)

$$K = \frac{e^3}{8\pi^2 c\varepsilon_0 m^2} \simeq 2{,}364.10^4 \ (K \text{ in rad.A.}m^2.k_g^{-1} \text{ for } \Omega \text{ in rad})$$
$$\simeq 1{,}354.10^6 \quad (K \text{ in °.A.}m^2.k_g^{-1} \quad \text{for } \Omega \text{ in °})$$



Propagation in ionosphere: 3. Polarization



Elevation angle 0°

Elevation angle 90°

Consequence: Circular polarisation used for satellite links up to few GHz !

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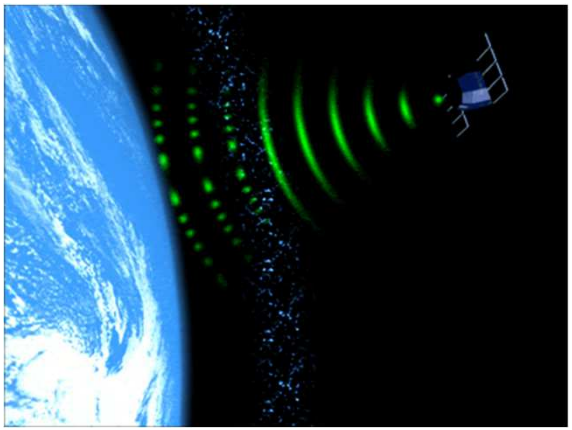
Propagation in ionosphere: 4. Scintillation

Scintillation:

Ionisation inhomogeneities create amplitude and phase variations.



Credit: University of Bath

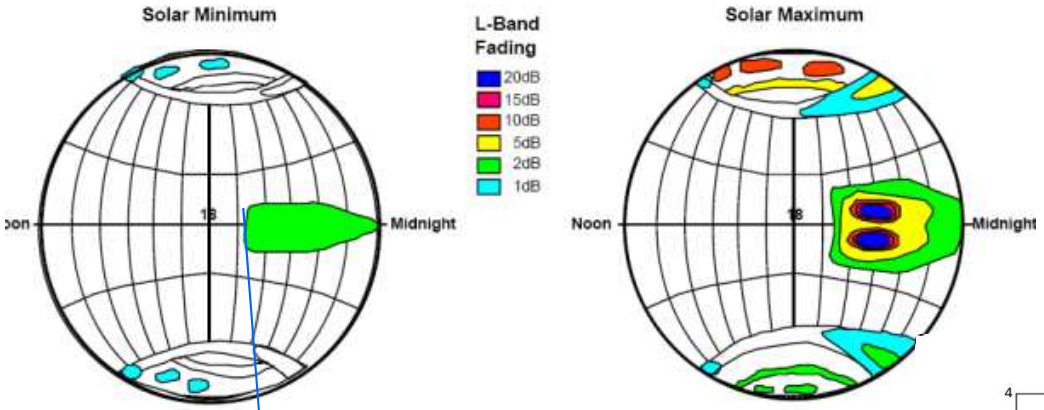
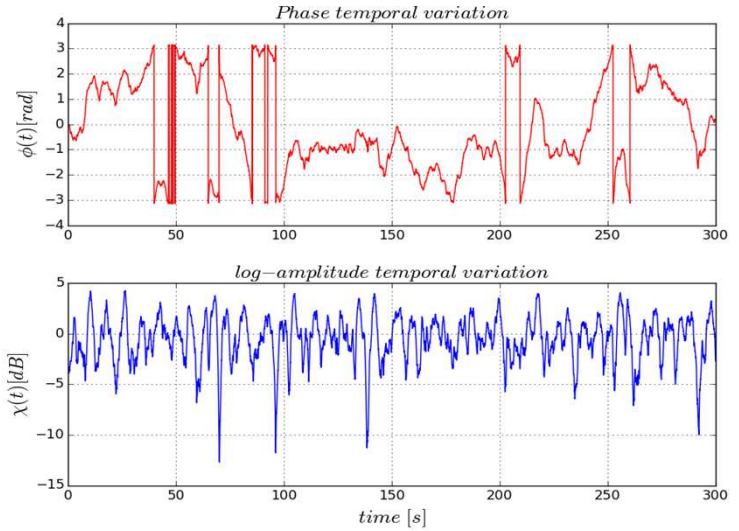


Figure from [Basu et al., 1988]

After sunset

Example of temporal variation :





Propagation in ionosphere: 4. Scintillation

Scintillation: 3 indices of scintillation characterization

- Amplitude : S_4 index [no units]

$$S_4 = \sqrt{\frac{\langle I^2 \rangle - \langle \mathrm{I} \rangle^2}{\langle \mathrm{I} \rangle^2}}$$

(I: signal intensity, I = Amplitude²)

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Propagation in ionosphere: 4. Scintillation

Scintillation: 3 indices of scintillation characterization

- Amplitude : S_4 index [no units]

$$S_4 = \sqrt{\frac{\langle I^2 \rangle - \langle \mathrm{I} \rangle^2}{\langle \mathrm{I} \rangle^2}}$$

At L band:

- S_4 < 0.3 weak
- 0.3 ≤ S_4 < 0.6 moderate
- 0.6 ≤ S_4 strong.

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Propagation in ionosphere: 4. Scintillation

Scintillation: 3 indices of scintillation characterization

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$$S_4 = \sqrt{\frac{\langle I^2 \rangle - \langle \mathrm{I} \rangle^2}{\langle \mathrm{I} \rangle^2}}$$

(I: signal intensity, I = Amplitude²)

- Phase : σ_φ index [rad]

$$\sigma_\varphi = \sqrt{\langle \varphi^2 \rangle - \langle \varphi \rangle^2}$$

standard deviation of the signal phase φ

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Propagation in ionosphere: 4. Scintillation

Scintillation: 3 indices of scintillation characterization

•Amplitude : S_4 index [no units]

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At L band:

- S_4 < 0.3 weak
- 0.3 ≤ S_4 < 0.6 moderate
- 0.6 ≤ S_4 strong.

•Phase : σ_φ index [rad]

$$\sigma_\varphi = \sqrt{\langle \varphi^2 \rangle - \langle \varphi \rangle^2}$$

- σ_φ < 0.25 rad, weak
- 0.25 < σ_φ < 0.5 rad moderate
- σ_φ > 0.5 rad. strong

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Propagation in ionosphere: 4. Scintillation

Scintillation: 3 indices of scintillation characterization

- Amplitude : S_4 index [no units]

$$S_4 = \sqrt{\frac{\langle I^2 \rangle - \langle I \rangle^2}{\langle I \rangle^2}}$$

(I: signal intensity, I = Amplitude²)

- Phase : σ_φ index [rad]

$$\sigma_\varphi = \sqrt{\langle \varphi^2 \rangle - \langle \varphi \rangle^2}$$

standard deviation of the signal phase φ

- ROTI : Rate Of change of **TEC** Index [TECu/mn]

$$ROTI = \sqrt{\langle ROT \rangle^2 - \langle ROT^2 \rangle}$$

with the Rate of change of TEC (ROT) defined by $ROT = \frac{\Delta STEC}{\Delta t}$

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Propagation in ionosphere: 4. Scintillation

Scintillation: 3 indices of scintillation characterization

- Amplitude : S_4 index

$$S_4 = \sqrt{\frac{\langle I^2 \rangle - \langle I \rangle^2}{\langle I \rangle^2}}$$

At L band:

- $S_4 < 0.3$ weak
- $0.3 \leq S_4 < 0.6$ moderate
- $0.6 \leq S_4$ strong.

- Phase : σ_φ index

$$\sigma_\varphi = \sqrt{\langle \varphi^2 \rangle - \langle \varphi \rangle^2}$$

- $\sigma_\varphi < 0.25$ rad, weak
- $0.25 < \sigma_\varphi < 0.5$ rad moderate
- $\sigma_\varphi > 0.5$ rad. strong

- ROTI : Rate Of change of TEC Index

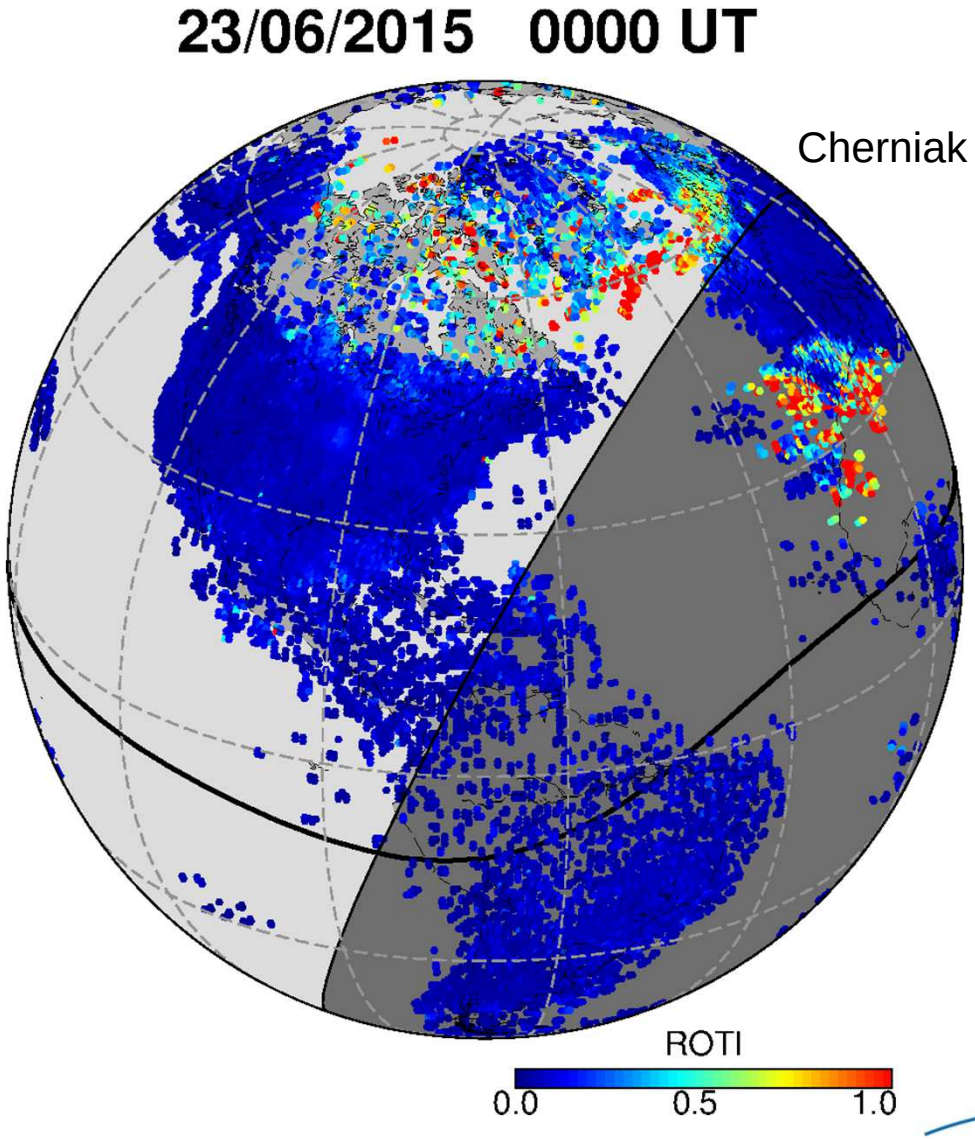
0 < ROTI < 1	"Low activity. No adverse effects expected."
1 ≤ ROTI < 3	"Normal activity. For most GNSS users, this level of activity will not cause problems. A slight increase in position error may be detected in high-accuracy applications."
3 ≤ ROTI < 5	"Moderate activity. GNSS users may have difficulty getting a good coordinate solution."
ROTI ≥ 5	"High activity. GNSS users will have difficulty getting a good coordinate solution. Network base stations may lose lock on satellites."



Propagation in ionosphere: 4. Scintillation

The June 2015 geomagnetic storm

24th solar cycle maximum



Cherniak et al, JGR, 2019

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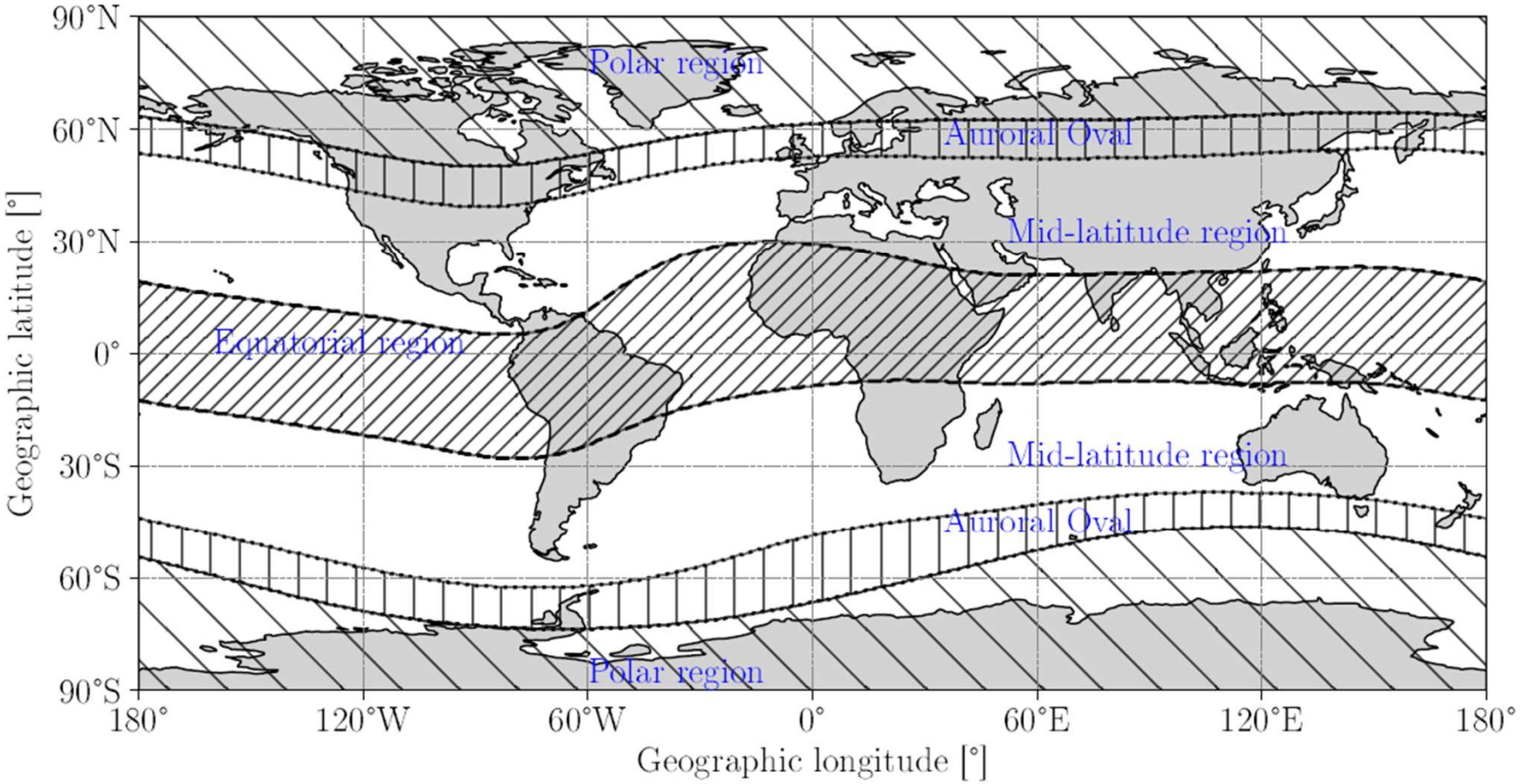
Propagation in ionosphere: 4. Scintillation



Different regions of ionosphere activity

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Propagation in ionosphere: 4. Scintillation

Synthetic Aperture Radar

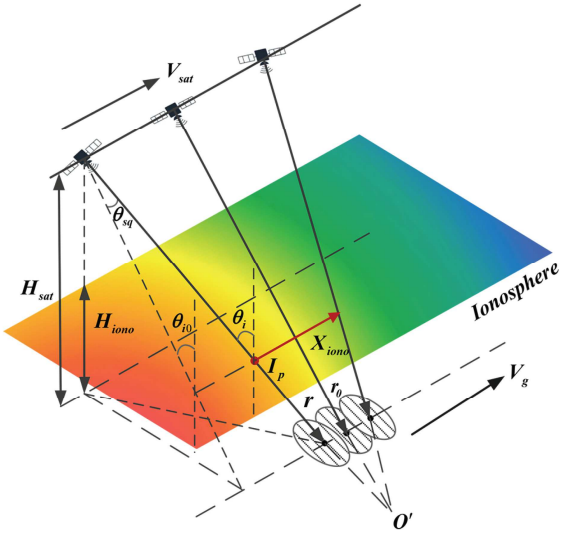

Figure – Observation geometry of a SAR system with ionosphere [Yu, 2019].

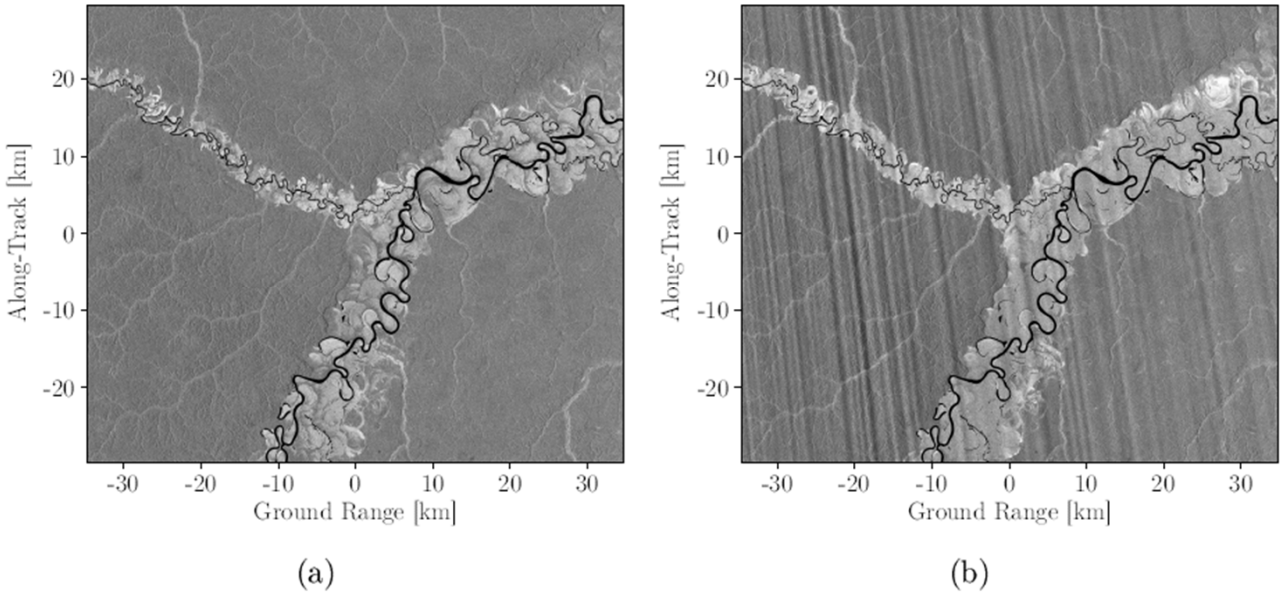

Figure – Two ALOS/PALSAR images over the amazon:
(a) not affected by scintillation and
(b) affected by scintillation

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Propagation in ionosphere: 4. Scintillation

Refraction index

$$n_\varphi(\vec{r},t) = \langle n_0 \rangle + n_1(\vec{r})$$

$\langle n_0 \rangle$ large scale component (=1)

$n_1(\vec{r})$ random turbulent component

3D Covariance function of the turbulent component of the refraction index

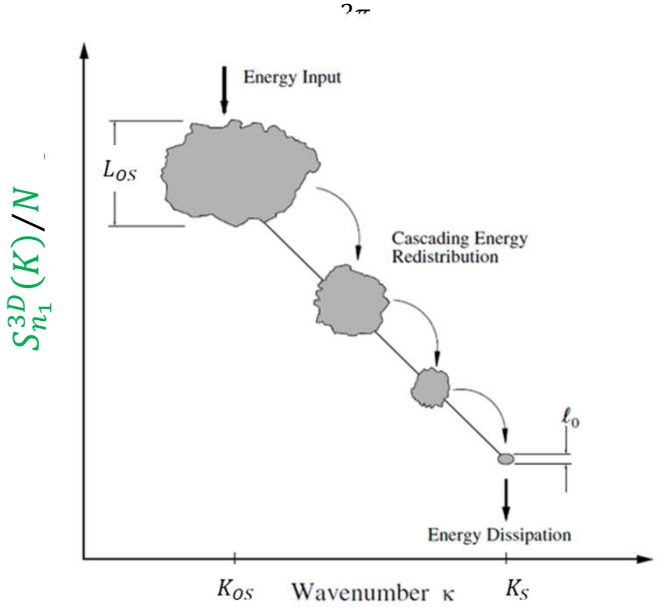

Figure : Inertial cascade phenomenon [Wheelon, 2004].

$$\langle n_1(\vec{r}) n_1(\vec{r'}) \rangle = \int_{-\infty}^{+\infty} dk^3 S_{n_1}^{3D}(K) e^{i\vec{k}(\vec{r}-\vec{r'})}$$

PSD

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Propagation in ionosphere: 4. Scintillation

Electron-density fluctuations are described by *Shkarofsky* [1968] spectrum:

$$S_{n_1}^{3D}(K) = S_{\Delta N_e}(K) = A_x A_y A_z C_s \left(A_x^2 K_{xH}^2 + A_y^2 K_{yH}^2 + A_z^2 K_{zH}^2 + K_0^2\right)^{-\frac{p}{2}}$$

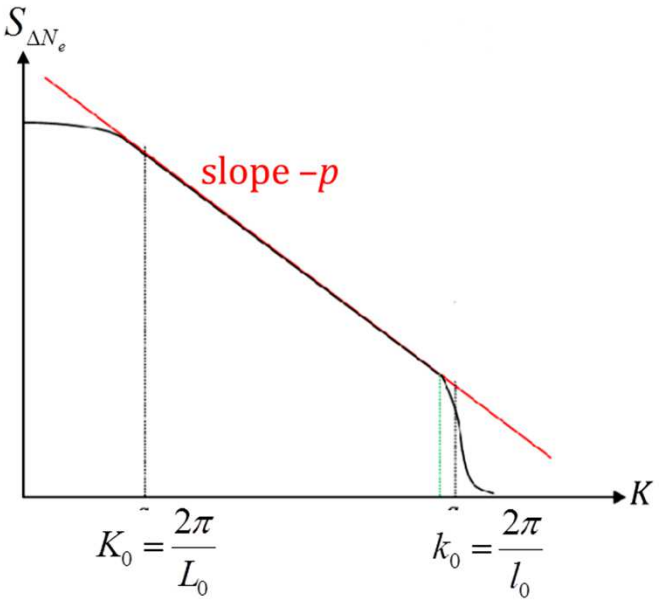

Figure : Spectrum of electron-density fluctuations

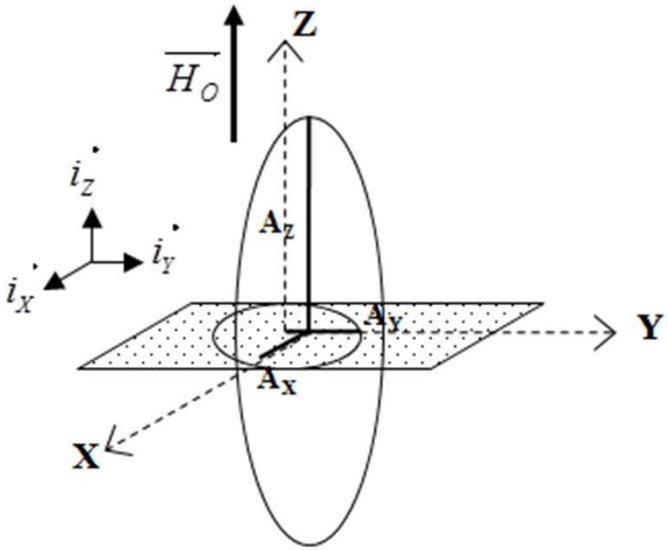

Figure : Ellipsoidal ionospheric irregularity With anisotropy ratios A_X,A_Y and A_Z elongated along the terrestrial magnetic field H_0

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Propagation in ionosphere: 4. Scintillation

Numerical modeling of scintillation

Helmholtz equation resolution (2D)

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}+k_0^2 n_\varphi^2\right)E=0$$

- With operator

$$Q=\sqrt{\frac{\partial^2}{\partial z^2}+k_0^2 n_\varphi^2} \quad \Longrightarrow \quad \left(\frac{\partial^2}{\partial x^2}+Q^2\right)E=0$$

- Factorisation

$$\left(\frac{\partial}{\partial x}+iQ\right)\left(\frac{\partial}{\partial x}-iQ\right)E=0$$

- Forward propagation only

$$\frac{\partial E}{\partial x}=iQE=i\left(\sqrt{\frac{\partial^2}{\partial z^2}+k_0^2 n_\varphi^2}\right)E.$$

- Refraction index approx $n_\varphi^2 \approx (1+2n_1)$
- [Feit and Fleick, 1978] approx: $Q \approx \sqrt{\frac{\partial^2}{\partial z^2}+k_0^2}+k_0 n_1.$

$$E(x+\delta x,z)=e^{iQ\delta x}E(x,z)$$



Propagation in ionosphere: 4. Scintillation

Numerical modeling of scintillation

$$E(x+\delta x,z) = e^{iQ\delta x}E(x,z) = e^{i\sqrt{\frac{\partial^2}{\partial z^2}+k_0^2}\,\delta x}\, e^{ik_0\delta x\, n_1(z)}E(x,z)$$

- Introducing a Fourier transform on z (Split Step Fourier scheme - SSF)

$$E(x+\delta x,z) = e^{ik_0\delta x\tilde{n}_1(z)}\, f_{1D}^{-1}\left(e^{ik_x\delta x} f_{1D}(E(x,z))\right)$$

$$k_x = \begin{cases} \sqrt{k_0^2 - k_z^2} & si\ |k_z| \le k_0 \\ i\sqrt{k_z^2 - k_0^2} & si\ |k_z| > k_0 \end{cases}$$

- By introducing the x-dependence of refractive index fluctuations through a phase term

$$\phi(z) = \delta x\, \tilde{n}_1(z) = \int_x^{x+\delta x} n_1(\xi,z)d\xi$$

$$E(x+\delta x,z) = e^{ik_0\phi(z)}\, f_{1D}^{-1}\left\{e^{ik_x\delta x} f_{1D}[E(x,z)]\right\}$$



Propagation in ionosphere: 4. Scintillation

Numerical modeling of scintillation

PWE approach: refraction effet modeling by MPS (Multiple Phase Screens)

- 2D : PWE-SSF 2D/MPS 1D

$$E(x + \delta x, z) = e^{ik_0\phi(z)}\, f_{1D}^{-1}\left\{e^{ik_x\delta x} f_{1D}[E(x, z)]\right\}$$

- 3D : PWE-SSF 3D/MPS 2D

$$E(x + \delta x, y, z) = e^{ik_0\phi(y,z)}\, f_{2D}^{-1}\left\{e^{ik_x\delta x} f_{2D}[E(x, y, z)]\right\}$$

$$\phi(y, z) = \int_{x_0}^{x_0+\delta x} n_1(\xi, y, z)\, d\xi$$

$$S_\phi^{2D}(k_y, k_z) = 2\pi\delta x\, S_{n_1}^{3D}(k_x = 0, k_y, k_z)$$

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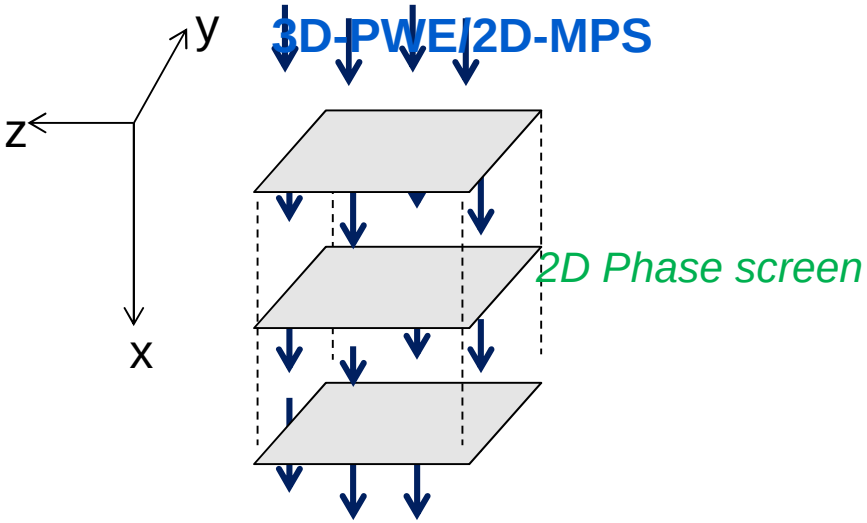
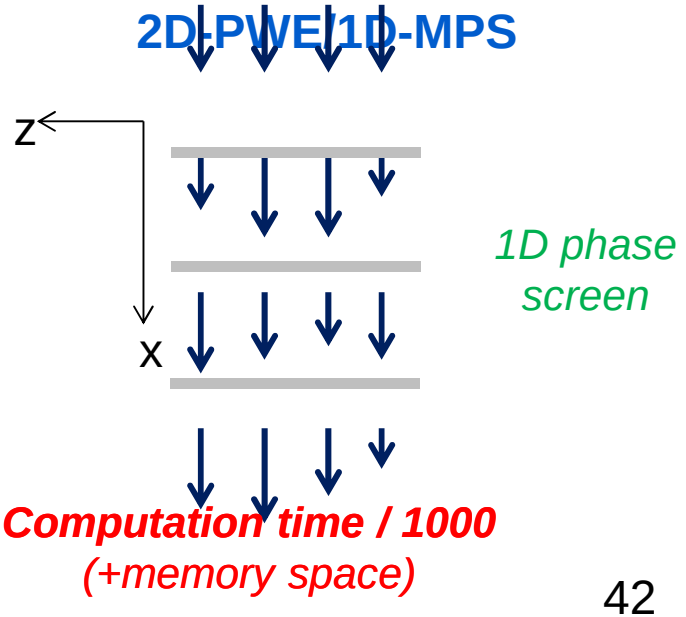
Propagation in ionosphere: 4. Scintillation

Numerical modeling of scintillation

PWE approach: refraction effet modeling by MPS (Multiple Phase Screens)

$$E(x + \delta x, y, z) = e^{ik_0\phi(y,z)}\, f_{2D}^{-1}\left\{e^{ik_x\delta x} f_{2D}[E(x, y, z)]\right\}$$





Computation time / 1000
(+memory space)

42

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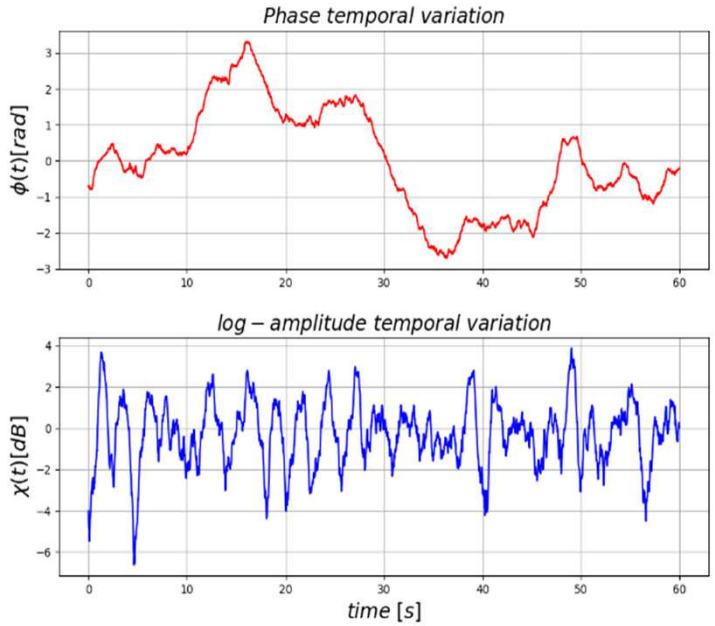
Propagation in ionosphere: 4. Scintillation

STIPEE: Software **T**ool for **I**onospheric **P**ropagation **E**ffects **E**valuation (ONERA/CNES)

STIPEE – PROP
(PWE-MPS numerical scheme)



STIPEE
PWE

$f = 1575\ MHz$

$\sigma_\phi = 0.24 rad.$

$S_4 = 0.35$

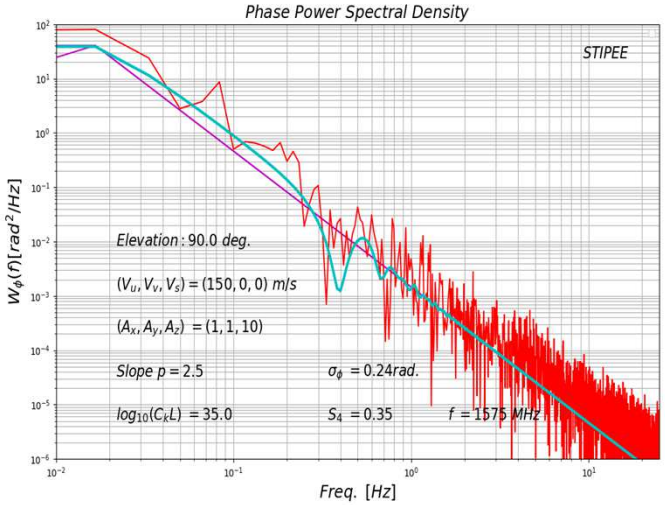
Elevation : 90.0 *deg.*

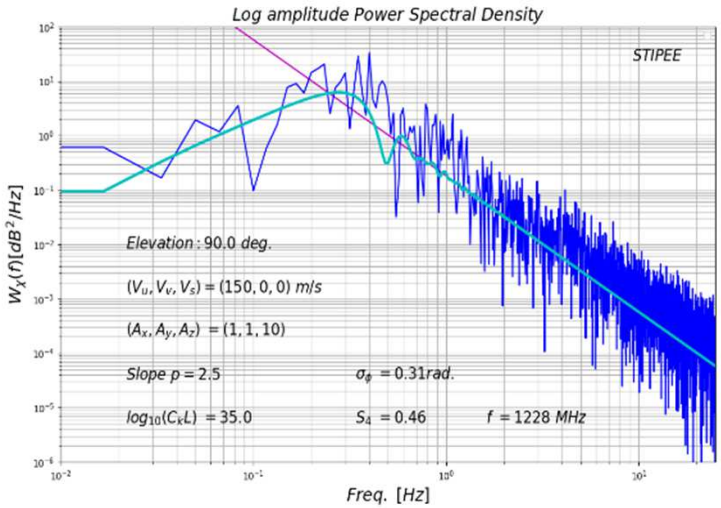
$(V_u, V_v, V_s) = (150, 0, 0)\ m$

$(A_x, A_y, A_z) = (1, 1, 10)$

Slope $p = 2.5$

$log_{10}(C_k L) = 35.0$







Propagation in ionosphere: 4. Scintillation

STIPEE: Software **T**ool for **I**onospheric **P**ropagation **E**ffects **E**valuation (ONERA/CNES)

STIPEE – PROP
(PWE-MPS numerical scheme)

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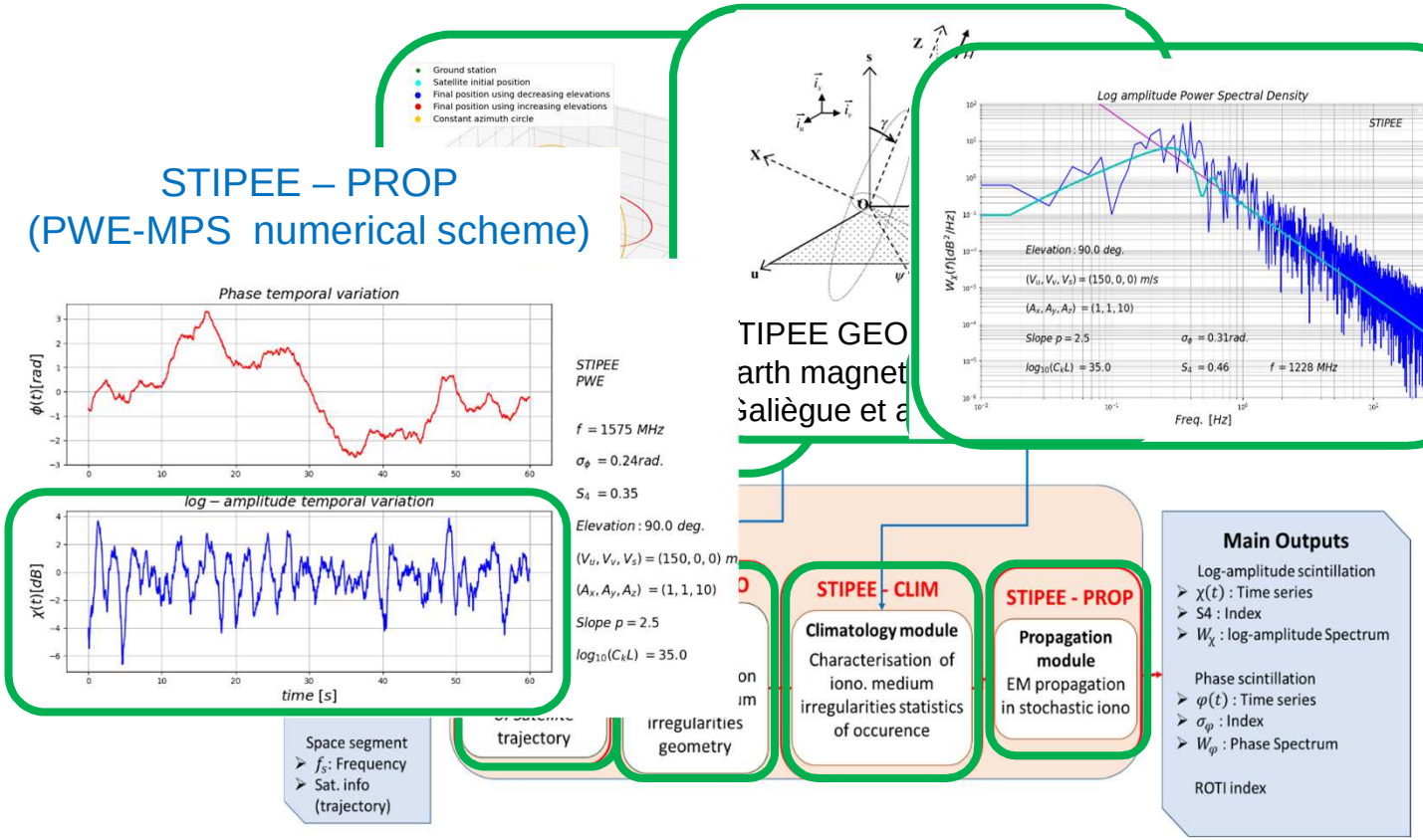

BLOCK DIAGRAM OF STIPEE MODEL FOR SCINTILLATION MODELING

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Propagation in ionosphere: 4. Scintillation

STIPEE code

A model of ionospheric irregularities climatology, compatible with the provider's scintillation tool and valid at all latitudes, so that the model is able to consider any type of Earth-satellite link.

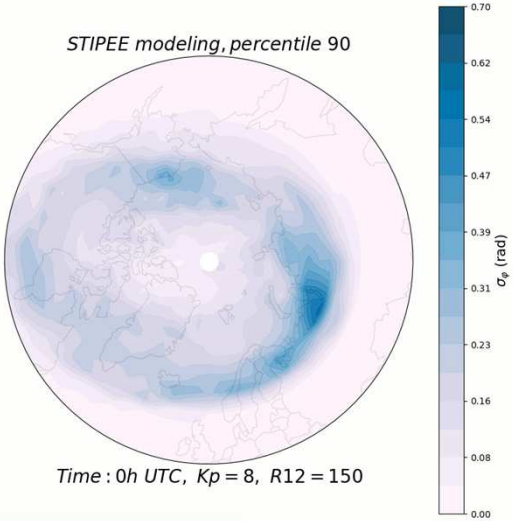

Figure: STIPEE modeling at high latitude predicting phase signal disturbances

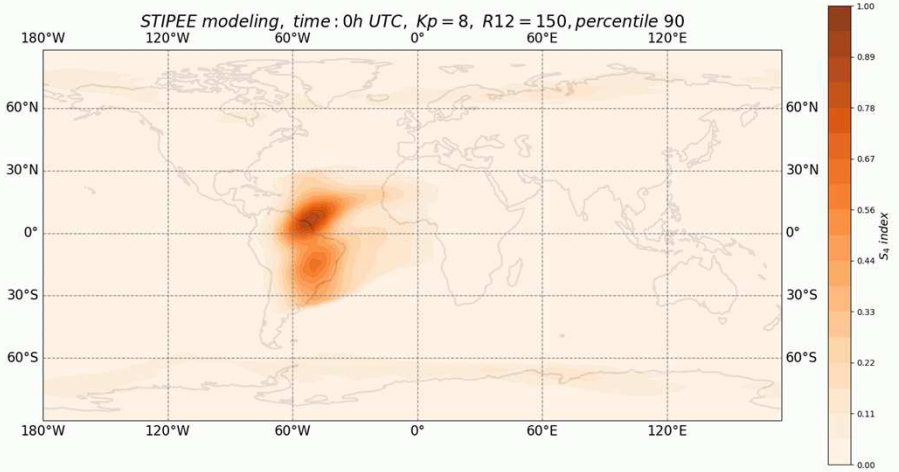

Figure: STIPEE at low latitude predicting amplitude signal disturbances

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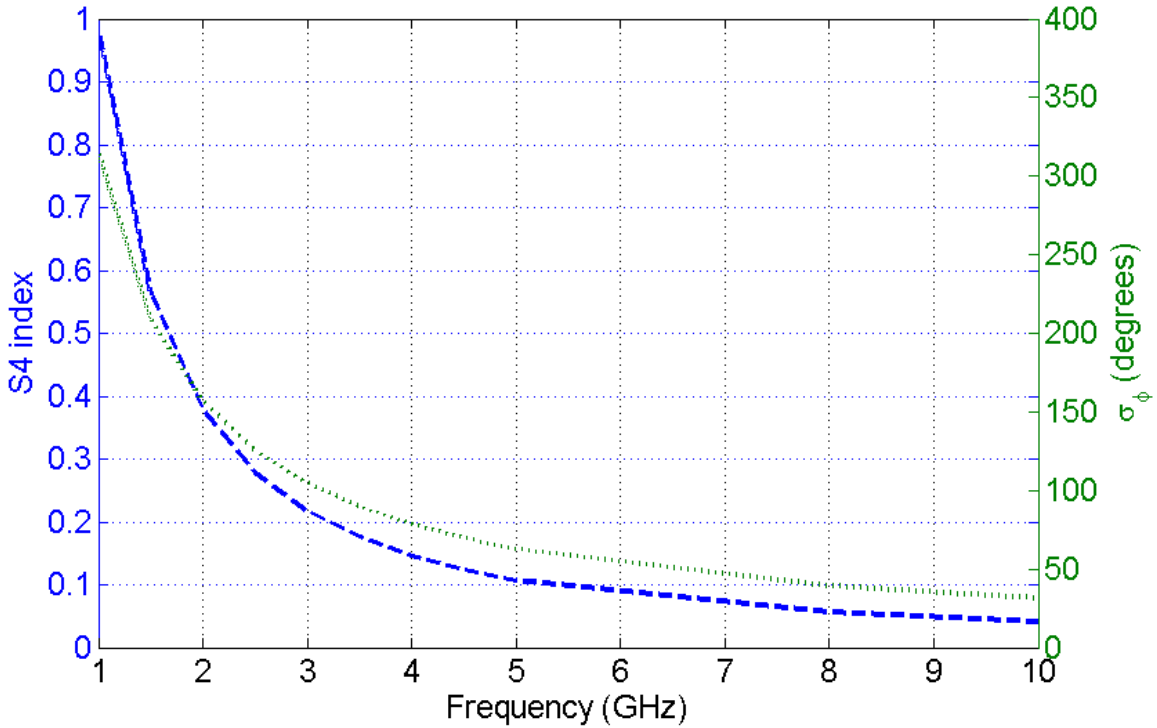
Propagation in ionosphere: 4. Scintillation

Frequency dependence:



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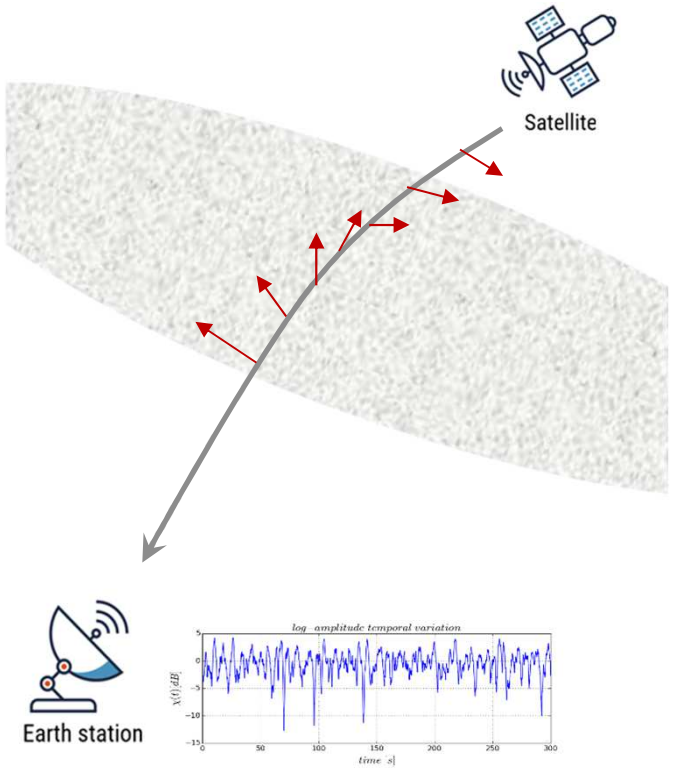
Ionosphere effects on earth-satellite links

Conclusion:

Several effects:

- Refraction
- Absorption/attenuation
- Faraday rotation
- Scintillation



Fade mitigation against ionospheric effects:

- Application of dual frequency techniques (such as GPS):
  propagation delay measure by difference.
- If single frequency, correction by Near-real-time TEC maps (provided by ESA for example swe.ssa.esa.int)

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Link by ionospheric refraction



Link by ionospheric refraction

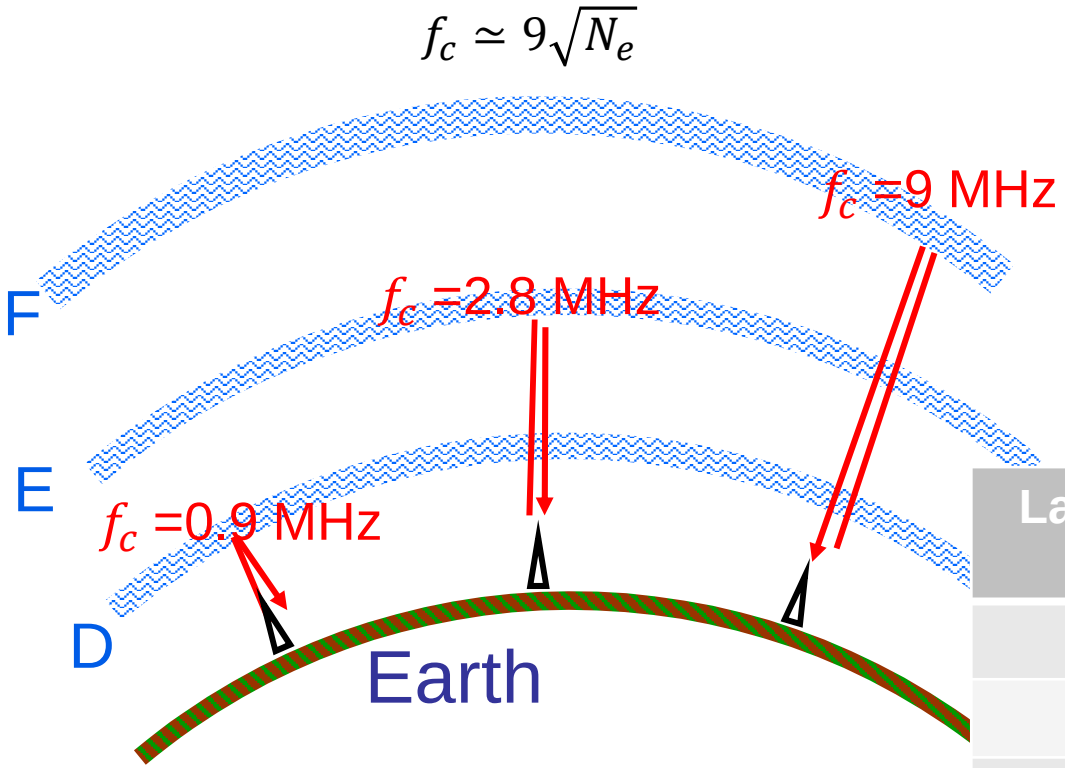
Why HF is special ?

At frequencies below the plasma frequency the ionosphere behaves as a refractive medium capable of returning radio waves toward the Earth.

This property is unique to the HF band (and lower frequency bands) and forms the basis of long-distance **skywave communications** and **over-the-horizon radar**.



Link by ionospheric refraction



Layer	Altitude [km]	N_e [e/m^3]	f_c [MHz]
D	60	10^{10}	0,9
E	150	10^{11}	2,8
F	300	10^{12}	9

Example of ionospheric layers

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Link by ionospheric refraction

Refraction:

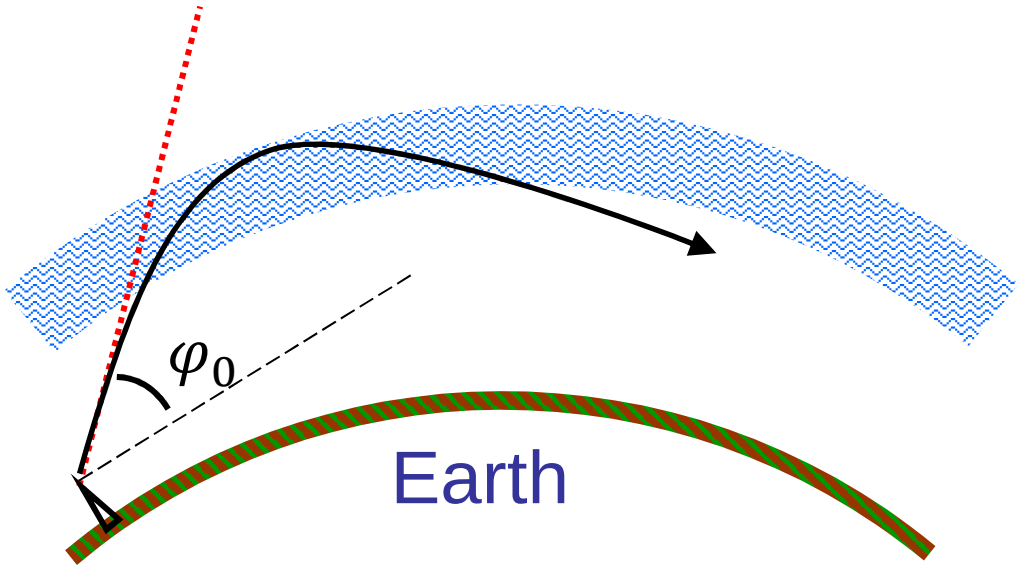
From Snell- Descartes law

$$n_\varphi(z_1)\cos(\varphi_1) = n_\varphi(z_2)\cos(\varphi_2)$$

for a stratified atmosphere

$$n_\varphi(z)\cos(\varphi) = \cos(\varphi_0)$$

with $n_\varphi = \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$



An Iterative computation
is possible across
the ionosphere layer

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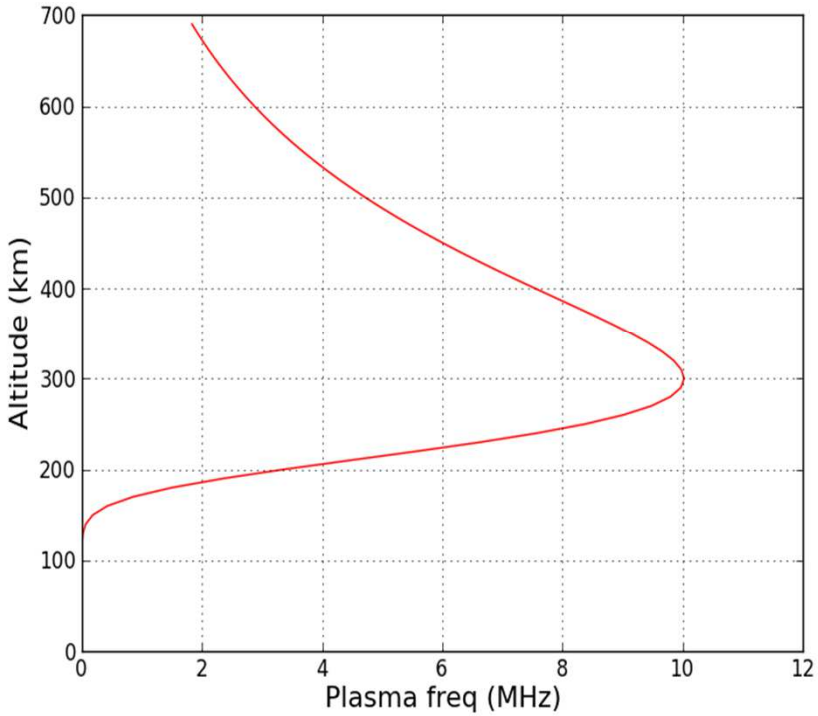
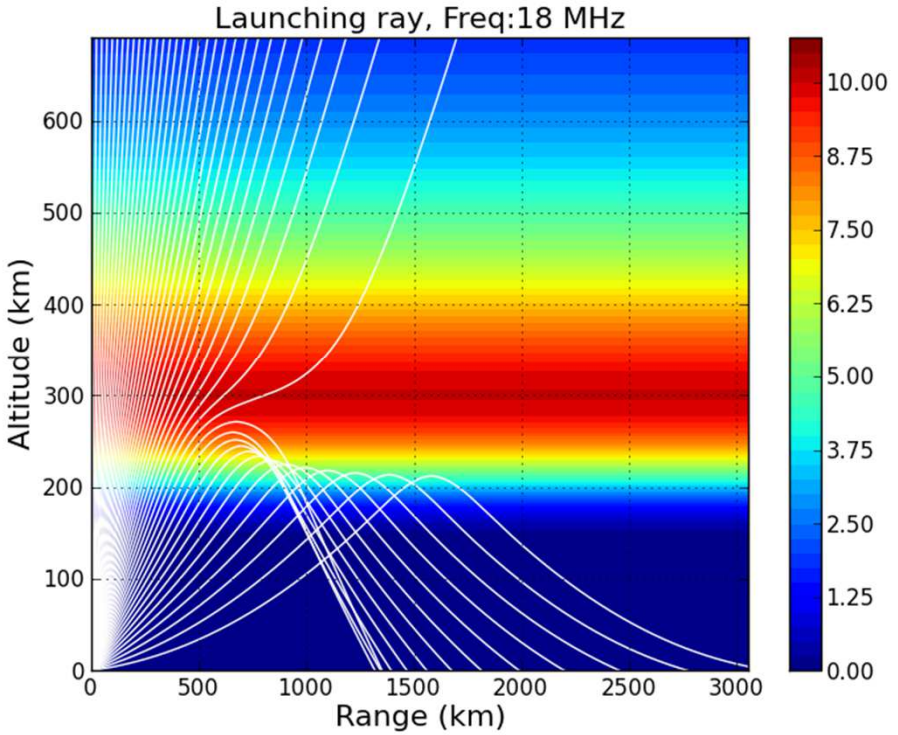
Link by ionospheric refraction

Example of launching ray in canonical ionosphere:



fc profile



fc map (color)

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Link by ionospheric refraction

Refraction:

At the reflection point $\varphi_R = 0$ ⇒ $$cos(\varphi_0) = \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

$$f \cos(\varphi_0) = \sqrt{f^2 - f_c^2} \Rightarrow$$

$$f^2 \cos^2(\varphi_0) = f^2 - f_c^2$$

$$f_c^2 = f^2 (1 - \cos^2(\varphi_0))$$

$$f_c = f \, sin(\varphi_0)$$

this is the « equivalent » vertical incidence frequency

with oblique incidence, the ionosphere reflects much higher frequencies than with normal incidence

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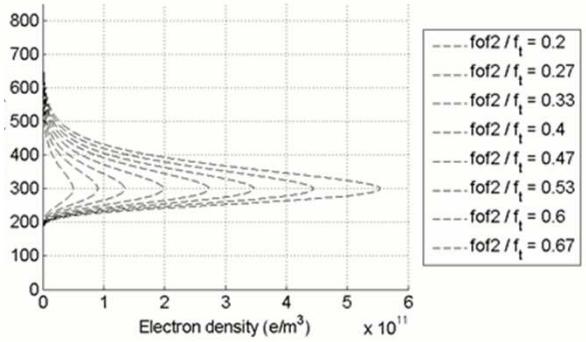
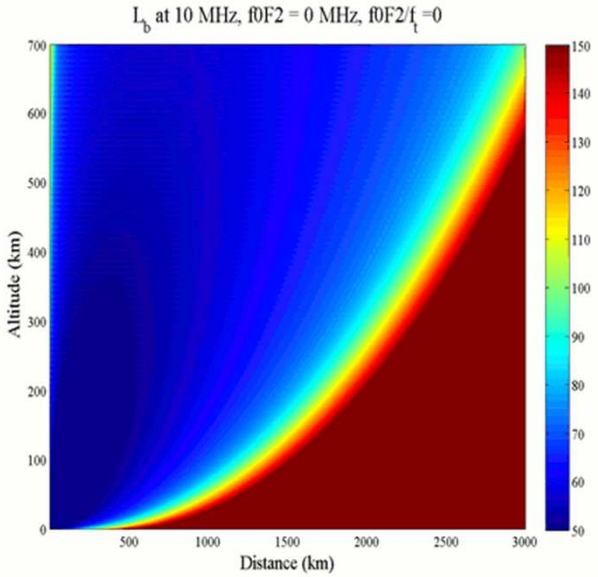
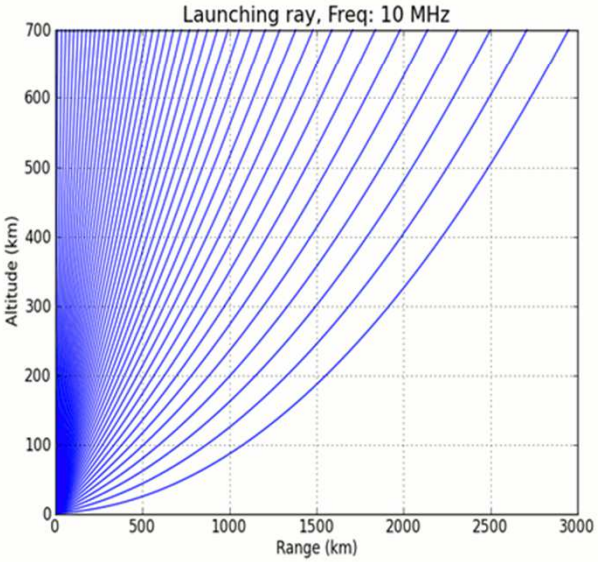
Link by ionospheric refraction

- Sky wave modeling







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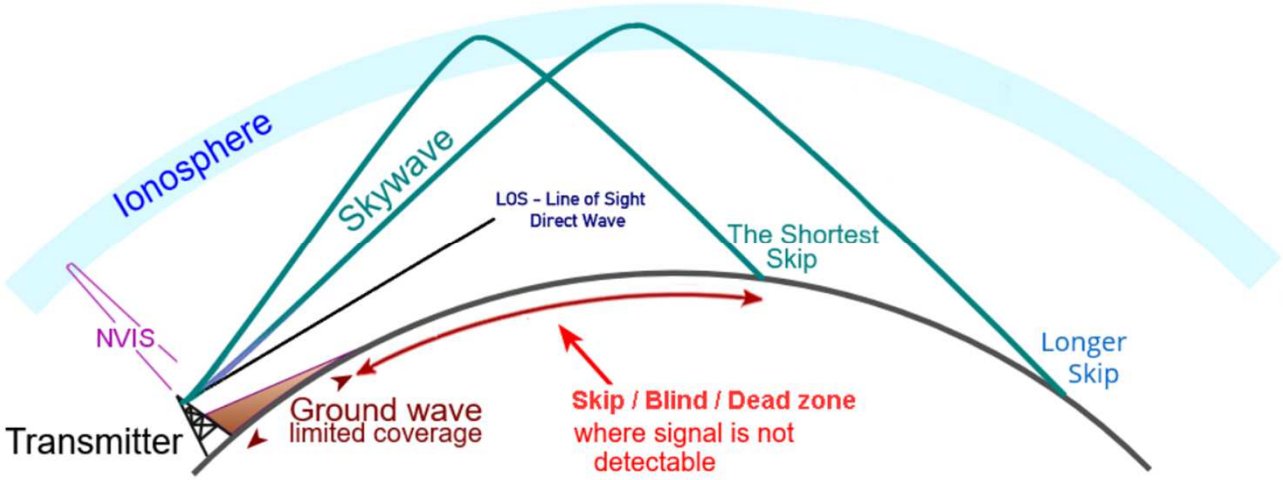
Link by ionospheric refraction



For a point to point HF radio-link:

LUF Lower Usable Frequency (function of signal to noise ratio)

MUF Maximum Usable Frequency (function of transmitted and iono plasma frequencies)

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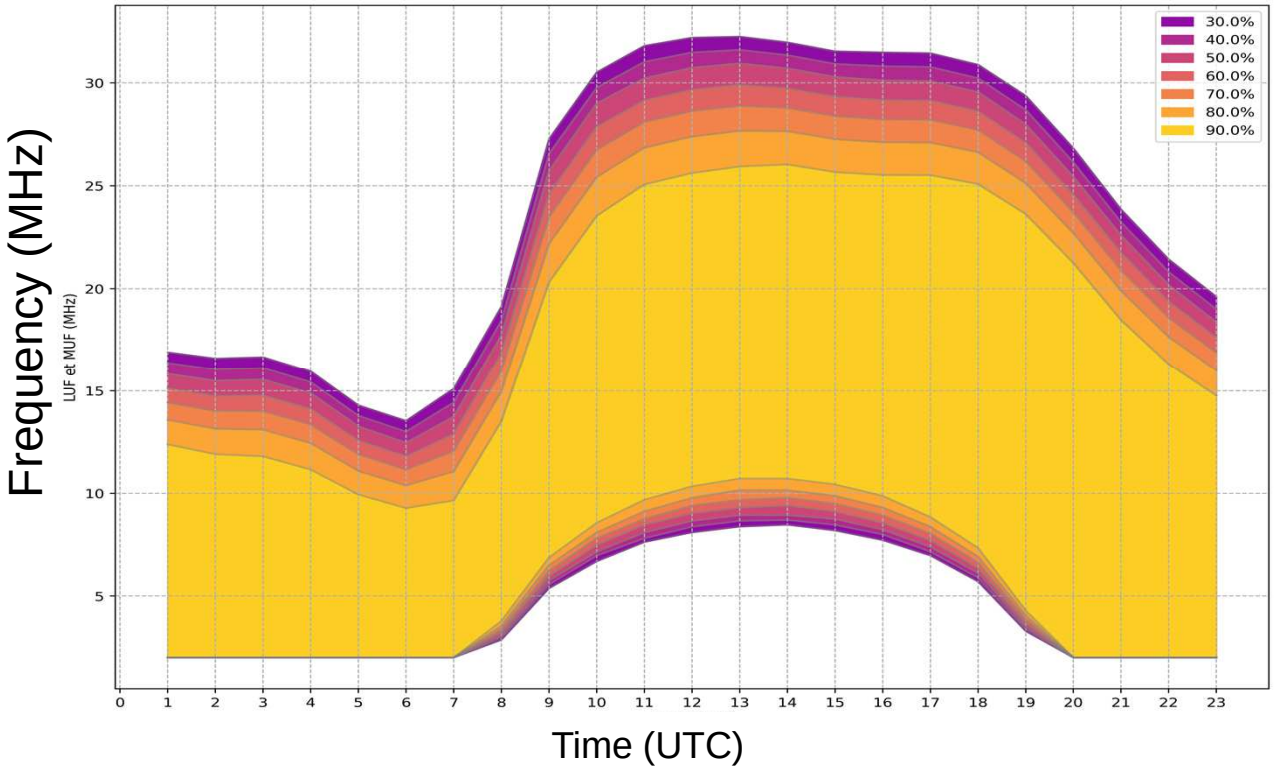
Link by ionospheric refraction

Variation of possible frequency for a HF link during the day



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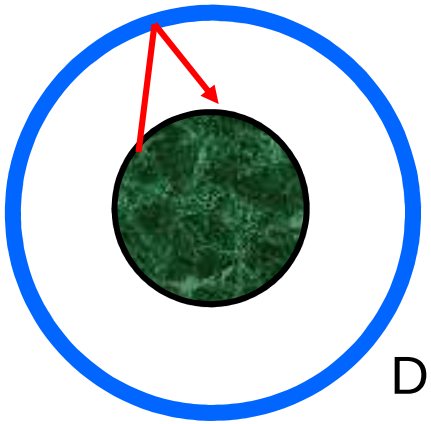
Link by ionospheric refraction

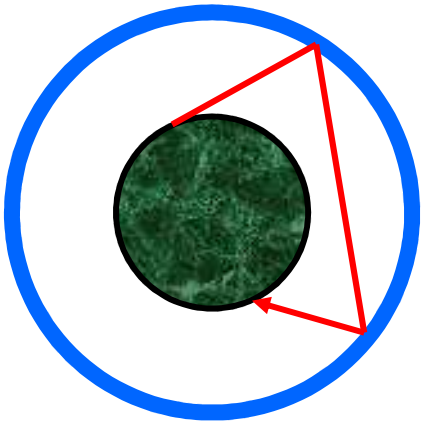
Multiple modes

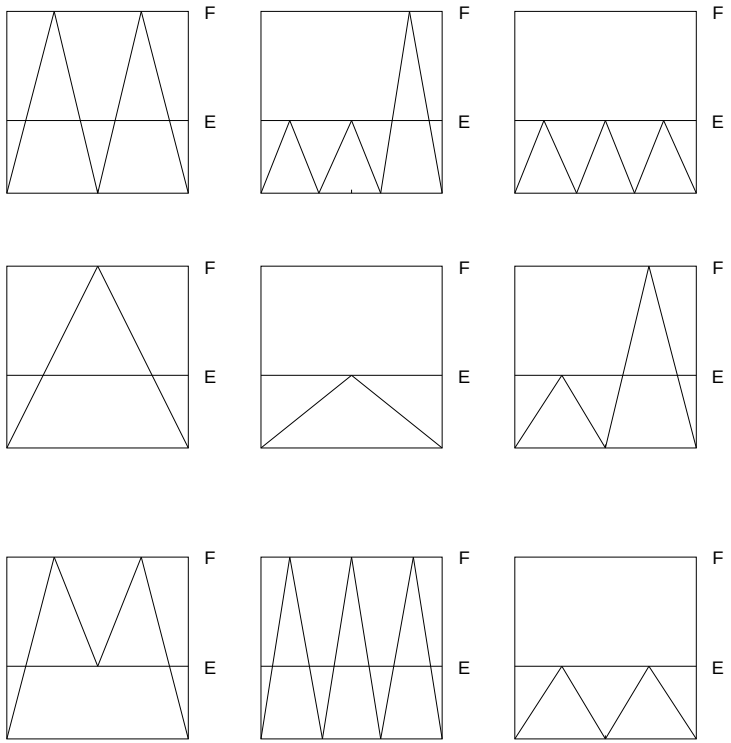
We can have a propagation with two reflections or more on layer E (2E), or F (as 3F), or hybrid (as 1F1E) ...

Simple reflection



Double reflection





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Ionosphere effect on propagation: refraction

Impact of Earth magnetic field

Equations with Earth magnetic field influence:

$\vec{V}$: particule speed
m : particule mass
$\vec{E}$: propagated electric field
$\vec{H}$: propagated magnetic field
ρ : volume charge
v : electron-neutral collision freq.
N_e : particules number per m^3
q : particule charge
$\vec{J}$: induced currents in ionized gaz

$$m\frac{d\vec{V}}{dt} + mv\vec{V} = q\vec{E} + q\left(\vec{V} \times \mu_0 \overrightarrow{H_T}\right) \quad (1)$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \varepsilon_0 \frac{d\vec{E}}{dt} \quad (2)$$

$$with \quad \begin{cases} \vec{J} = \rho\vec{V} = N_e q\vec{V} \\ \overrightarrow{H_T} = \overrightarrow{H_O} + \vec{H} \simeq \overrightarrow{H_O} \end{cases}$$

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Ionosphere effect on propagation: refraction

General formulation (Happleton-Hartree)

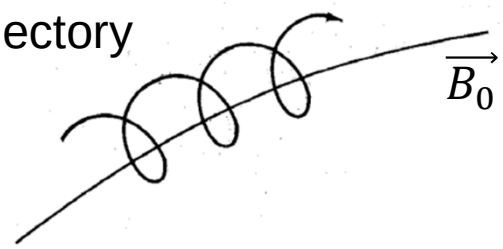
$$\boldsymbol{n_{\varphi}^{2}}(f) = 1 - \frac{X}{1 - jZ - \frac{Y_T^2}{2(1-X)} \pm \sqrt{\left(\frac{Y_T^2}{2(1-X)}\right)^2 + Y_L^2}}$$

with:

- $Y_T^2 = Y^2 sin^2\theta$
- $Y_L^2 = Y^2 cos^2\theta$
- $Z = \nu/\omega$
- $X = \omega_c^2/\omega^2$

$$\boldsymbol{Y} = \frac{\boldsymbol{\omega_H}}{\boldsymbol{\omega}}$$ With the cyclotron angular frequency $\boldsymbol{\omega_H}$: $$\boldsymbol{\omega_H} = \frac{|\boldsymbol{e}|.B_0}{m}$$

Electron trajectory



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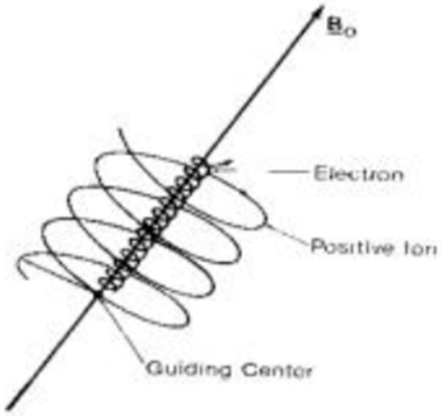
Ionosphere effect on propagation: refraction

Cyclotron angular frequency (or gyrofrequency):

Movement of a charged particle in Earth magnetic field $\overrightarrow{B_0}$

Lorentz force = centrifugal force

$$q\vec{v} \wedge \overrightarrow{B_0} = . e . v_\perp . B_0 = \frac{m . \omega_H^2}{r_H}$$



	Larmor radius	Gyrofrequency
Electrons	2 cm	1,2 MHz
Ions (O+)	4 m	50 Hz

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Ionosphere effect on propagation: refraction

General formulation (Happleton-Hartree)

$$n_{\varphi}^2(f) = 1 - \frac{X}{1 - jZ - \frac{Y_T^2}{2(1-X)} \pm \sqrt{\left(\frac{Y_T^2}{2(1-X)}\right)^2 + Y_L^2}}$$

with:

- $Y_T^2 = Y^2 sin^2\theta$
- $Y_L^2 = Y^2 cos^2\theta$
- $Z = \nu/\omega$
- $X = \omega_c^2/\omega^2$

If $Z = \frac{\nu}{2\pi f} \ll 1$ (typically $\nu \leq 1\ MHz$)

$$n_{\varphi}^2(f) = 1 - \frac{2X(1-X)}{2(1-X) - Y_T^2 \pm \sqrt{Y_T^4 + 4Y_L^2(1-X)^2}}$$

And if $Y \ll 1$,

$$n_{\varphi}^2(f) \simeq (1 - X) \simeq \left(1 - \frac{f_c^2}{f^2}\right)$$

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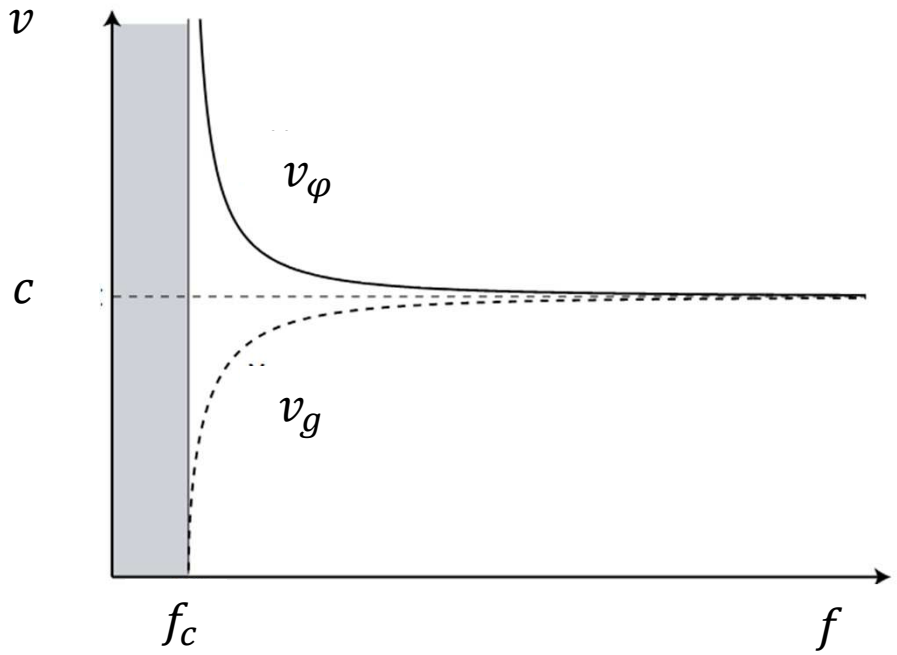
Ionosphere effect on propagation: refraction

$Velocities: simplified\ index$

$$Phase: \ v_\varphi = \frac{\omega}{k} = \frac{c}{n_\varphi}$$

$$Group: \ v_g = \frac{d\omega}{dk} = \frac{c}{n_g} \qquad with \qquad n_\varphi \approx \left[1 - \left(\frac{f_c}{f}\right)^2\right]^{1/2} \Rightarrow \ v_\varphi = \frac{c}{\sqrt{1 - X}}$$

$$\Rightarrow \ v_g = c\sqrt{1 - X}$$



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Ionosphere effect on propagation : refraction

$Velocities:\ with\ Earth\ magnetic\ field$

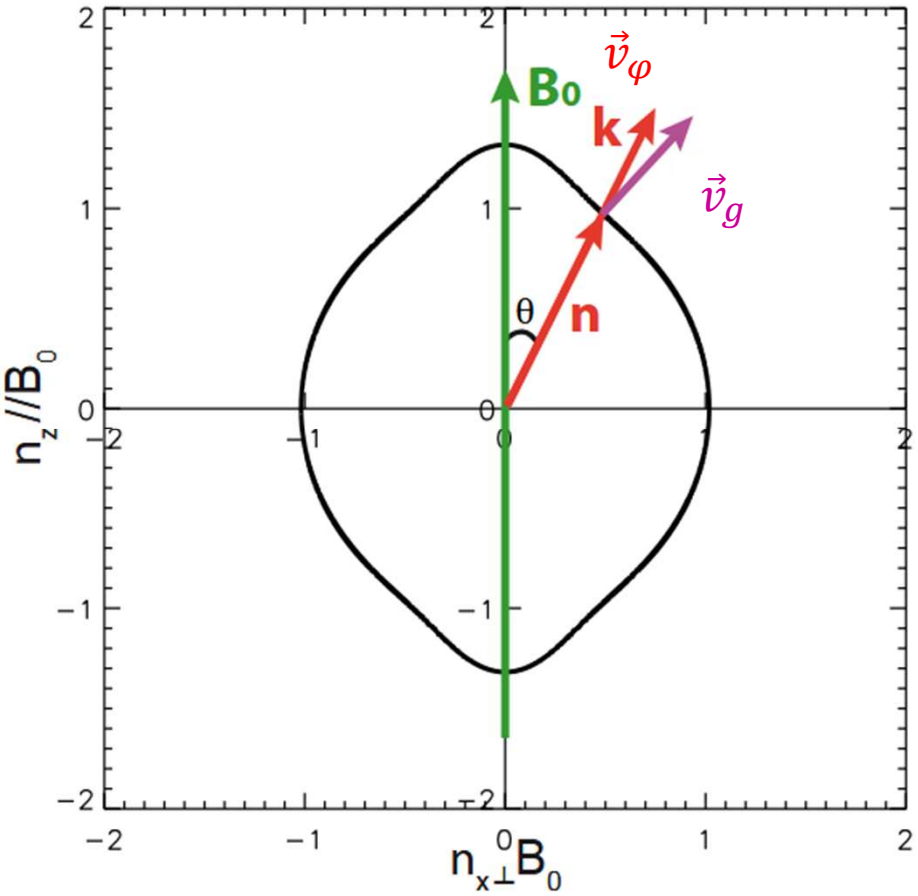
$$Phase: \vec{v}_\varphi = \frac{\omega}{k}\vec{k}$$

The refractive index vector $\vec{n}$ and wave vector $\vec{k}$ are collinear.

$$Group: \vec{v}_g = \vec{\nabla}_k \omega \qquad with \quad \vec{k} = \omega\vec{n}/c$$

For a given θ angle, the group velocity vector $\vec{v}_g$ is normal to the refraction index surface [Gautier, 2014]

Figure: Phase index surface from Appleton-Hartree in (n_x, n_z) plane, for X = 0.81 et Y² = 4.41, with B_0 along e_z axis.



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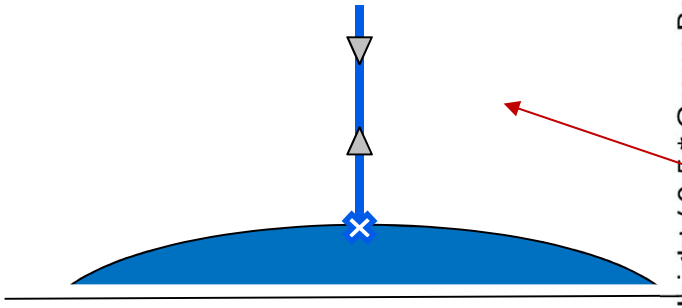
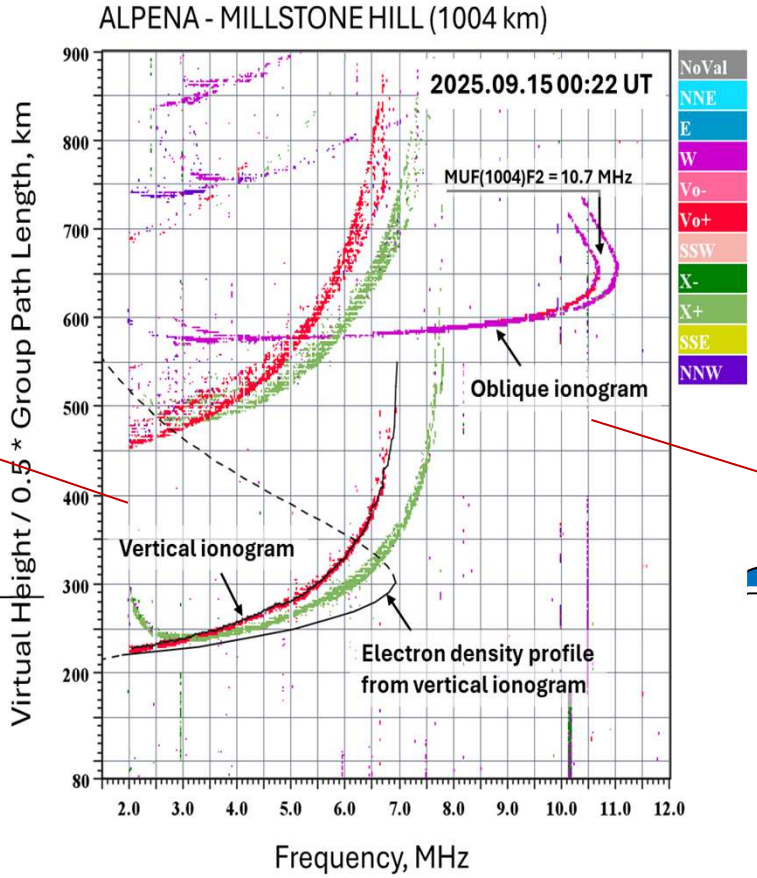
Ionosphere effect on propagation : refraction

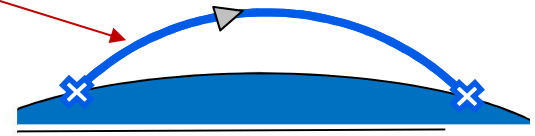
Measured Data

Vertical soundings





Oblique soundings



$$f_x - f_o \approx \frac{f_H}{2}$$

$$f_H = \frac{\omega_H}{2\pi}$$

Figure: Example of vertical and oblique ionograms [From Galkin et al.]

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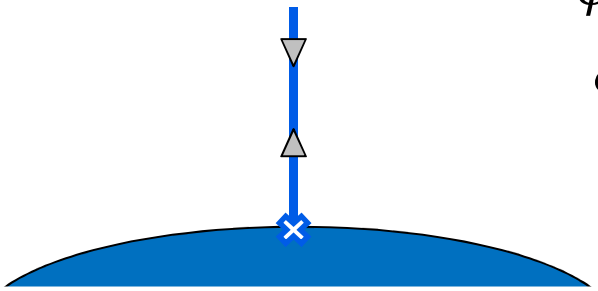
Ionosphere effect on propagation : refraction

Vertical sounding



$$\varphi_0 = \frac{\pi}{2}; \quad \varphi_R = 0 \Rightarrow cos(\varphi_0) = 0 = n_\varphi\,(f)\,cos(\varphi_R)$$

$$0 = n_\varphi(f)$$

Two solutions :

$$n_\varphi^2(f) = 1 - \frac{2X(1-X)}{2(1-X) - Y_T^2 \pm \sqrt{Y_T^4 + 4Y_L^2(1-X)^2}}$$

Ordinary $(+) \Rightarrow X = 1$

Extraordinary $(-) \Rightarrow \mathrm{X} = 1 \pm \mathrm{Y}$

Note: In general, on frequencies greater than the electron gyrofrequency ($Y < 1$) the extraordinary wave does not normally penetrate to the $X = 1 + Y$ level

$$(+) \Rightarrow X = 1 \quad \Rightarrow f_c^2/f_o^2 = 1 \quad f_c^2 = f_o^2 \quad f_o = \sqrt{f_x^2 - \mathrm{f_H f_x}}$$

$$(-) \Rightarrow \mathrm{X} = 1 - \mathrm{Y} \quad \Rightarrow f_c^2/f_x^2 = 1 - \mathrm{f_H/f_x} \quad \Rightarrow f_c^2 = f_x^2 - \mathrm{f_H} f_x \quad \approx f_x\left(1 - \frac{\mathrm{f_H}}{2\mathrm{f}_x}\right)$$

$$f_x - f_o \approx \frac{f_H}{2}$$

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Ionosphere effect on propagation : refraction

Summary :

- General solution to characterize the medium :
  Happleton-Hartree equation (dispersion relation)
- Plasma is anisotropic due to Earth magnetic field influence
- Induces 2 separate solutions, commonly called **O**rdinary ('+') and **Ex**traordinary ('-')

How to compute propagation: Ray approaches (iterative process with range)

- by solving Happleton-Hartree equation (numerical approach)
- by simplifying refractive index:
  - Snell - Descartes law applied to multilayer ionosphere
  - Decomposition into series of Parabolic layers and solving analytically



Ionosphere effect on propagation : refraction

Model solving Haselgrove equations

- **Basic principle:** Hamiltonian formulation of the wave equation -> coupled equation systems giving the trajectory and wave vector of the wave at each point of the trajectory.
- **Initial parameters:** frequency, azimuth, elevation -> require a search method to reach the receiver.
- **Numerical resolution methods:** Runge-Kutta-4 or Runge-Kutta-Dormand-Prince to better control the integration error.
- **Limitation:** Assume a slow variation of the ionosphere -> unsuitable for studying short-lived phenomena such as bubbles.

$$H = \frac{1}{2}\Re\left[\left(\frac{c}{\omega}\right)^2\left(k_r^2 + k_\theta^2 + k_\varphi^2\right) - n_\varphi^2\right]$$

$$\begin{cases} \dfrac{dr}{dP} = -\dfrac{1}{c}\dfrac{dH}{dk_r}\Big/\dfrac{dH}{d\omega} \\ \dfrac{d\theta}{dP} = -\dfrac{1}{rc}\dfrac{dH}{dk_\theta}\Big/\dfrac{dH}{d\omega} \\ \dfrac{d\varphi}{dP} = -\dfrac{1}{rc\sin(\theta)}\dfrac{dH}{dk_\varphi}\Big/\dfrac{dH}{d\omega} \end{cases}$$

$$\begin{cases} \dfrac{dk_r}{dP} = \dfrac{1}{c}\dfrac{dH}{dr}\Big/\dfrac{dH}{d\omega} + k_\theta\dfrac{d\theta}{dP} k_\varphi\sin(\theta)\dfrac{d\varphi}{dP} \\ \dfrac{dk_\theta}{dP} = \dfrac{1}{r}\left(\dfrac{1}{c}\dfrac{dH}{d\theta}\Big/\dfrac{dH}{d\omega} - k_\theta\dfrac{dr}{dP} + k_\varphi r\cos(\theta)\dfrac{d\varphi}{dP}\right) \\ \dfrac{dk_\varphi}{dP} = \dfrac{1}{r\sin(\theta)}\left(\dfrac{1}{c}\dfrac{dH}{d\varphi}\Big/\dfrac{dH}{d\omega} - k_\varphi\sin(\theta)\dfrac{dr}{dP} - k_\varphi r\cos(\theta)\dfrac{d\theta}{dP}\right) \end{cases}$$

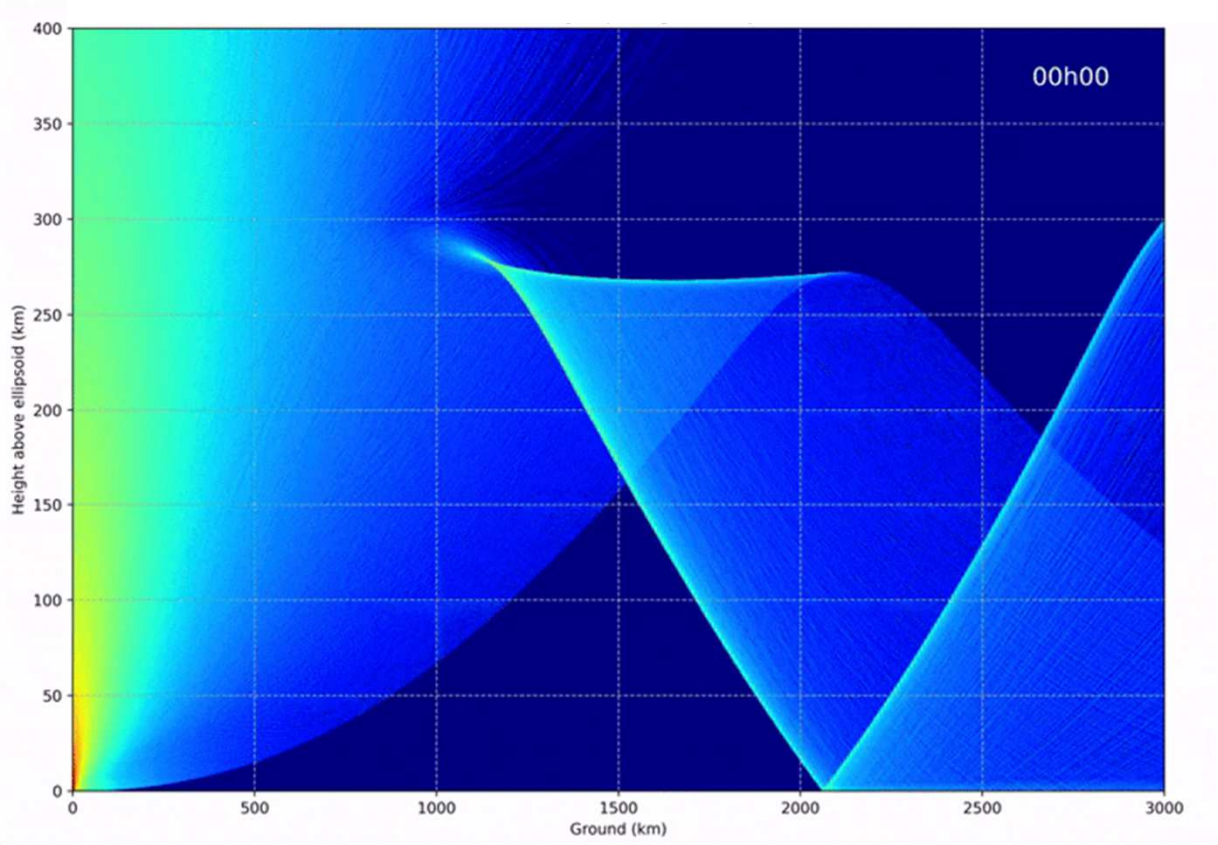
For more details, see *[M. Jones, J.J. Stephenson, A versatile three dimensional ray tracing computer program for radio waves in the ionosphere, U.S. Department of Commerce, Office of Telecommunications, Oct. 1975]*



Ionosphere effect on propagation : refraction

Different types of propagation models proposed:

Model solving Haselgrove equations



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Ionosphere effect on propagation : refraction

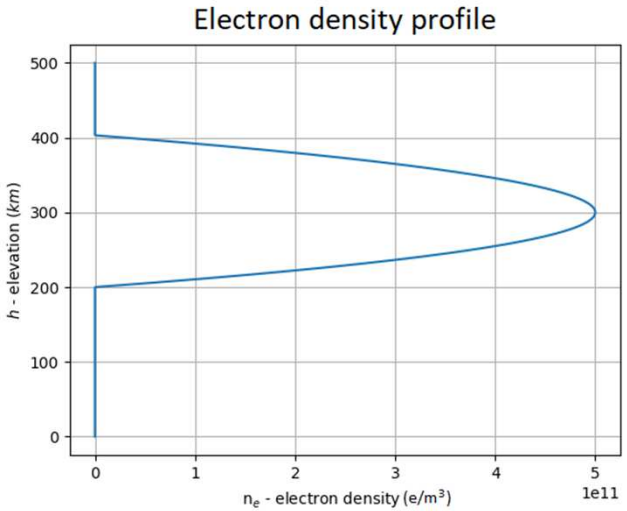
MQP model (Multi Quasi Parabolic)

- **Basic principle:** Description of vertical ionospheric density profile by segments of quasi-parabola. Then, analytic formulations available to compute the ray trajectory (i.e. group distance).

$$f(r)^2 = f_{Pi}^2 \left[1 \mp \frac{(r - r_{mi})^2}{y_{mi}^2} \frac{r_{bi}^2}{r^2} \right]$$

- **Initial parameters:** frequency, azimuth, elevation -> require a search method to reach the receiver.

- **Limitations:**
  - Approximation of electron density profile by MQP
  - No variability of the profile vs distance, nor lateral dimension
  - Assumes approximation of the refractive index



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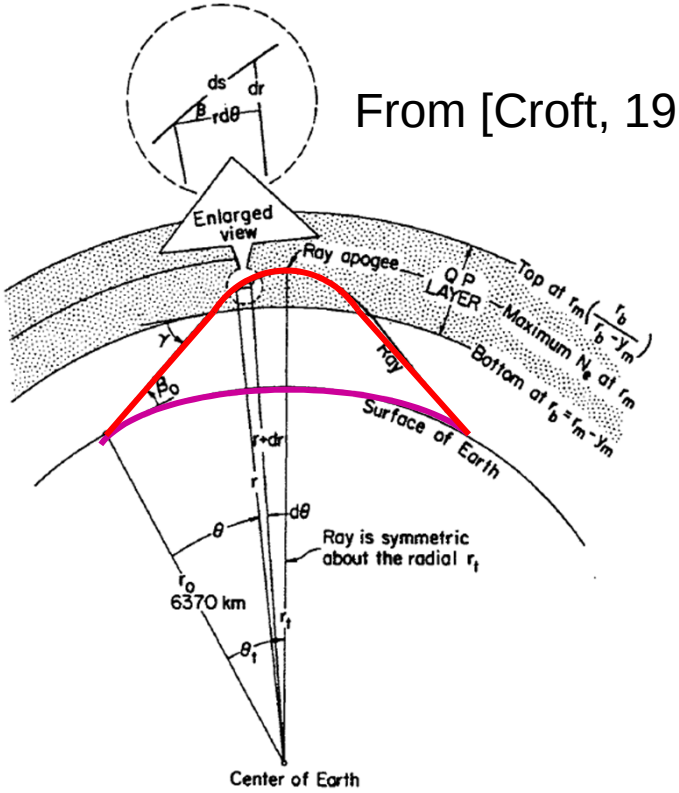
Ionosphere effect on propagation : refraction

MQP model (Multi Quasi Parabolic)

From [Croft, 1968]



Ground path

$$D = 2r_0 \int_0^{\theta_{max}} d\theta = 2r_0 \int_{r_0}^{r_{max}} \frac{dr}{r\,tan\beta} = 2\int_{r_0}^{r_{max}} \frac{r_0^2 cos\ \beta_0 dr}{r\sqrt{r^2 n_\varphi^2 - r_0^2 cos^2\ \beta_0}}$$

Group path

$$P' = 2\int_{r_0}^{r_{max}} \frac{ds}{n_\varphi} = 2\int_{r_0}^{r_{max}} \frac{dr}{n_\varphi \sin\beta} = 2\int_{r_0}^{r_{max}} \frac{r\,dr}{\sqrt{r^2 n_\varphi^2 - r_0^2 cos^2\ \beta_0}}$$

For each layer, the refractive index is defined by:

$$\begin{cases} N_{ei}(r) = a_i \mp b_i \left(1 - \frac{r_{mi}}{r}\right)^2 \\ a_i = N_{mi} \\ b_i = N_{mi}\left(\frac{r_{bi}}{y_{mi}}\right)^2 \end{cases}$$

where the electron density profile is described by a Quasi-Parabolic form

Then D and P' can be computed analytically

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Ionosphere effect on propagation : refraction

MQP model (Multi Quasi Parabolic)

Series of layers connected by junction layers (with **quasi parabolic shape**)

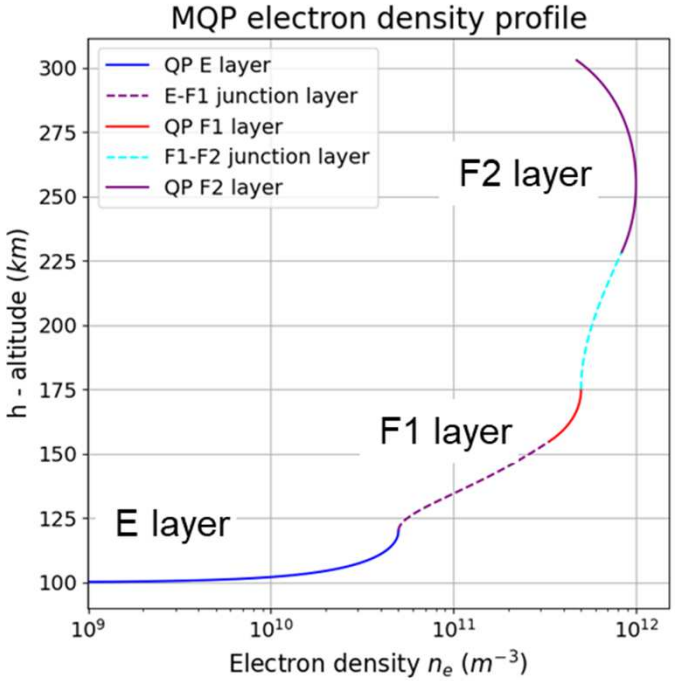

Figure: Ne vertical profile

Minus for i-th layer
plus for junction layer

$$\begin{cases} N_{ei}(r) = a_i \mp b_i \left(1 - \frac{r_{mi}}{r}\right)^2 \\ a_i = N_{mi} \\ b_i = N_{mi} \left(\frac{r_{bi}}{y_{mi}}\right)^2 \end{cases}$$

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Ionosphere effect on propagation : refraction

MQP model (Multi Quasi Parabolic)

Series of layers connected by junction layers (with **quasi parabolic shape**)

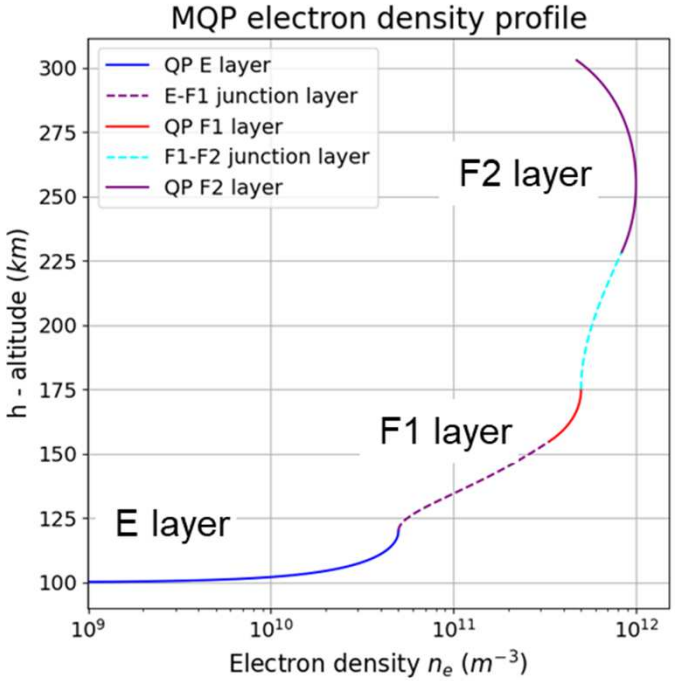

Figure: Ne vertical profile

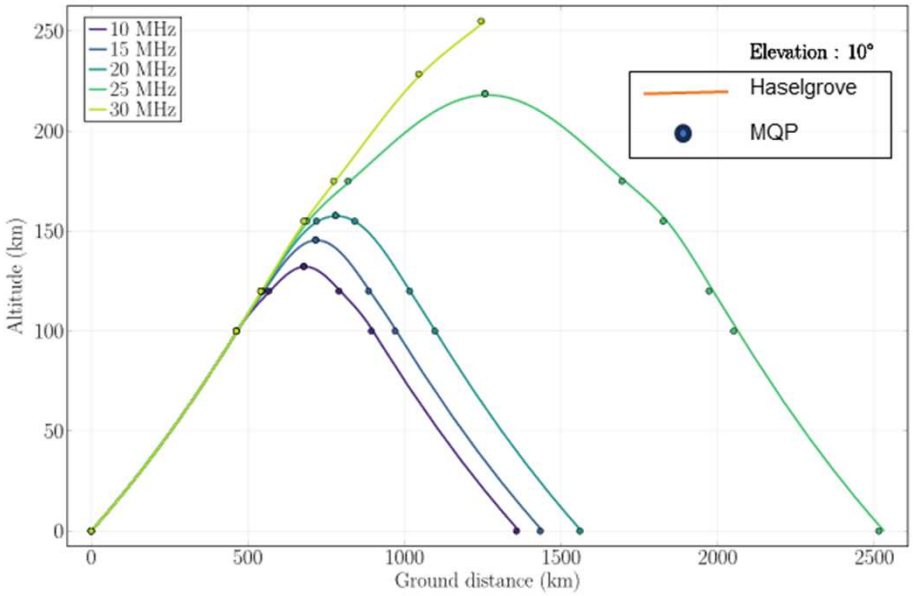

Figure: comparison of rays trajectory

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Ionosphere effect on propagation : refraction

MQP model (Multi Quasi Parabolic)

Series of layers connected by junction layers (with **Quasi Parabolic shape**)

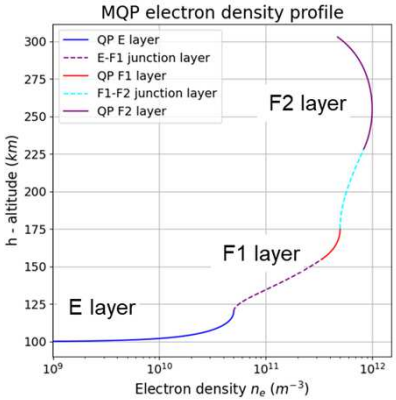
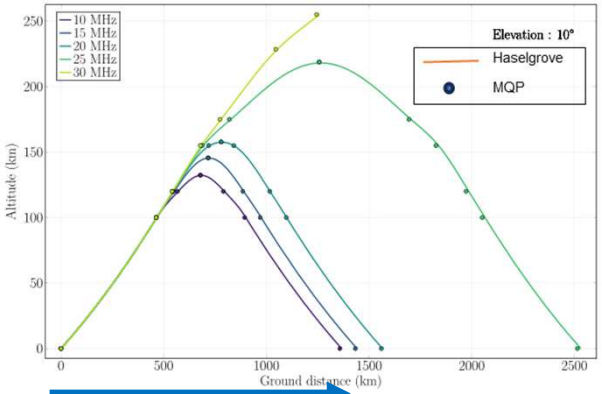


Figure: Ne vertical profile



D_{MQP}

Figure: comparison of rays trajectory

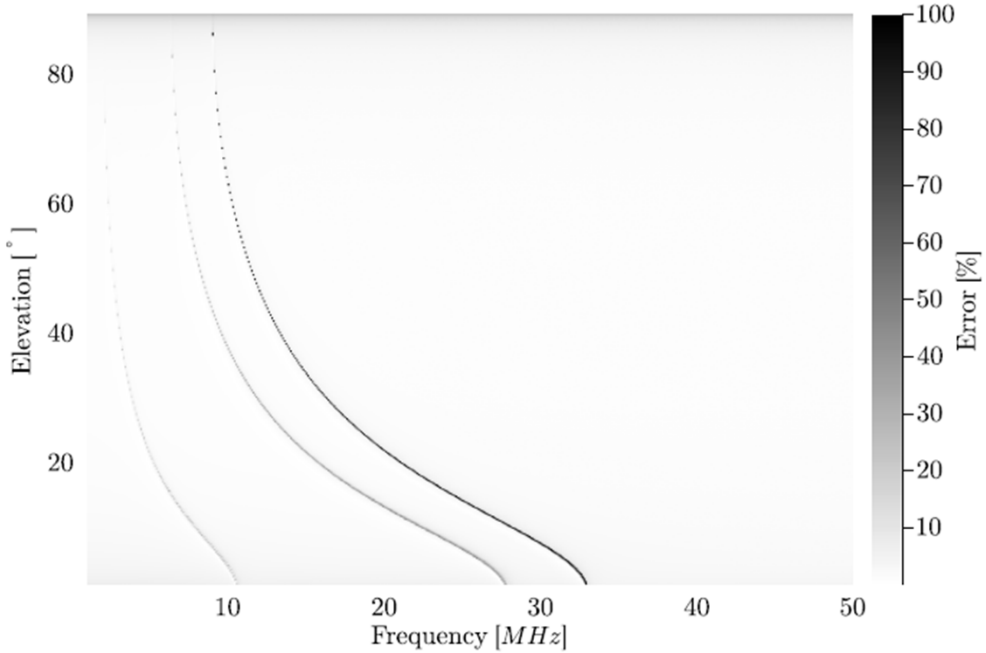


Figure: Comparison of rays trajectory : difference of ground distance

$$e = \frac{|D_{MQP} - D_{Haselgrove}|}{D_{MQP}}$$

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Link by ionospheric refraction

Conclusion:

Comparison of propagation modeling techniques

	MQP	Numerical resolution of Haselgrove equations
Electron density profile	- (but can be improved with high number of QP sements)	+++
Computation time	+++	+
Range variability	-	+++
Earth magnetic field	-	+++

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Summary of propagation effects

Frequency	Ionophere dominant Effects	Examples of impacted systems
VLF/ LF 3–30 kHz / 30–300 kHz	(Earth-iono. guide) D-region absorption	Submarine Communication, Longwave Radio Broadcasting
HF 3–30 MHz	Refraction (skywave), attenuation, Faraday	HF radio communications / Over the Horizon Radars
VHF 30–300 MHz	Refraction, scintillation	AIS (Automatic Identification System) with ships, Weather Satellite Downlinks (NOAA-LEO)
UHF (L-band) 300 MHz – 3 GHz	Delay, scintillation	Positioning systems (GNSS), SAR systems
S to X-band 2-12 GHz	Scintillation	SAR systems

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Appendices

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Appendix: propagation in a dispersive medium

Maxwell equations:

$$\vec{\nabla} \times \vec{E} = -i\omega\mu_0 \vec{H} \quad \rightarrow \quad \vec{H} = \frac{\vec{\nabla} \times \vec{E}}{-i\omega\mu_0}$$

$$\vec{\nabla} \times \vec{H} = i\omega n_\varphi^2 \epsilon_0 \vec{E} \quad \rightarrow \quad \vec{\nabla} \times \left(\vec{\nabla} \times \vec{E}\right) = -\Delta\vec{E} = k^2\vec{E}$$

$$= \omega^2 n_\varphi^2 \mu_0 \epsilon_0 \vec{E} = \frac{\omega^2}{c^2} n_\varphi^2 \vec{E}$$

Dispersion equation : $k^2 = \frac{\omega^2}{c^2} n_\varphi^2$

$$with \quad n_\varphi^{\,2} = 1 - \left(\frac{\omega_c}{\omega}\right)^2 \quad \rightarrow \quad \boxed{kc = \sqrt{\omega^2 - \omega_c^2}} \qquad \boxed{k^2c^2 = \omega^2 - \omega_c^2}$$

Phase speed: $v_\varphi = \frac{\omega}{k} = \frac{c}{n_\varphi}$

$$\boxed{2k\,dk\,c^2 = 2\omega d\omega}$$

Group speed: $v_g = \frac{d\omega}{dk} = \frac{c}{n_g}$ $\qquad \boxed{n_g = \frac{1}{n_\varphi}}$

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Appendix : Constants definition

Ionospheric delay

$$\begin{cases} \text{particule charge : } q = -e = 1{,}602 \times 10^{-19} A.s \\ \text{particule mass : } m = 9{,}109 \times 10^{-31} kg \\ \text{vacuum permittivity : } \varepsilon_0 = 8{,}854 \times 10^{-12} k_g^{-1} m^{-3} s^4 A^2 \end{cases}$$

$$f_c = \frac{\omega_c}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{N_e e^2}{m\,\varepsilon_0}} \simeq 8{,}97\sqrt{N_e} \rightarrow m^{-3/2}$$

$$8{,}97\sqrt{N_e} \rightarrow m^{3/2}.Hz$$

$$n_\varphi = \sqrt{1 - \frac{f_c^2}{f^2}} \simeq 1 - \frac{f_c^2}{2f^2} \simeq 1 - \frac{40{,}23\,N_e}{f^2}$$

$$40{,}23\,N_e \rightarrow m^3.Hz^2$$

$$\frac{40{,}23\,N_e}{f^2} \rightarrow m^{-3}$$

$$f^2 \rightarrow Hz^2$$

$$\delta l_{iono} = \int (n_g - 1)\,\mathrm{ds} \simeq \frac{40{,}23}{f^2}\int N_e\,\mathrm{ds} \rightarrow m^{-2}$$

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Appendix : Constants definition

Faraday rotation

B_0 : Earth magnetic field (Teslat)
N_e: electronic density (e/m³)
ϕ : angle between propagation wave vector and B_0 (rad)

$$\Omega = \frac{K}{f^2}\int_s N_e\, B_0 \cos(\phi)\, ds = \frac{K}{f^2}\int_h N_e\, B_0 \cos(\phi)\frac{dh}{sin\varphi}$$

$$K = \frac{e^3}{8\pi^2 c\varepsilon_0 m^2} \simeq 2{,}364.\,10^4 \text{ (in } \mathrm{rad.\,A.}\,m^2.\,k_g^{-1} \text{ for } \Omega \text{ in rad)}$$

$$\simeq 1{,}354.10^6 \quad \text{(in } °.\mathrm{A.}\,m^2.\,k_g^{-1} \quad \text{for } \Omega \text{ in °)}$$

$$\Omega = \frac{K}{f^2}\int_h N_e\, B_0 \cos(\phi)\frac{dh}{sin\varphi} \simeq \frac{K}{f^2} B_{av}\, STEC$$

With B_{av} average value of Earth magnetic field component parallel to wave vector.

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Appendix : Constants definition

Ionospheric Attenuation

$$|E| = \left|E_0 e^{i\omega t - i k_0 n_\varphi r}\right| = |E_0| e^{-\alpha r}$$

$$n_\varphi^2 = \left(1 - \frac{N_e e^2}{m\varepsilon_0}\frac{1}{\omega^2+\nu^2}\right) - i\frac{N_e e^2}{\omega m\varepsilon_0}\frac{\nu}{\omega^2+\nu^2}$$

$$n_\varphi \simeq 1 - i\frac{N_e e^2}{2\omega m\varepsilon_0}\frac{\nu}{\omega^2+\nu^2} \quad \textit{(refraction neglected, high frequency)}$$

$$\alpha = k_0 \frac{N_e e^2}{2\omega m\varepsilon_0}\frac{\nu}{\omega^2+\nu^2} = \frac{N_e e^2}{2m\, c\, \epsilon_0}\left(\frac{\nu}{\omega^2+\nu^2}\right)$$

$$\alpha \simeq \frac{e^2}{2mc\epsilon_0}\left(\frac{N_e\nu}{\omega^2}\right) = 4{,}6.\,10^{-2}\left(\frac{N_e\nu}{\omega^2}\right) dB/km$$

$$\alpha \simeq \frac{e^2}{2mc\epsilon_0(2\pi)^2}\left(\frac{N_e\nu}{f^2}\right) = 1{,}16.\,10^{-3}\left(\frac{N_e\nu}{f^2}\right) dB/km$$

$$\textit{Generalisation :}\ A_{dB} = \frac{1{,}16.10^{-6}}{f^2}\int_s N_e(s)\nu(s)\, ds$$

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