

DISPEC

Scientific exploitation of space Data for improved Ionospheric SPECification

L1.2 session 2

Data processing using the Discrete Fourier Transform (DFT)

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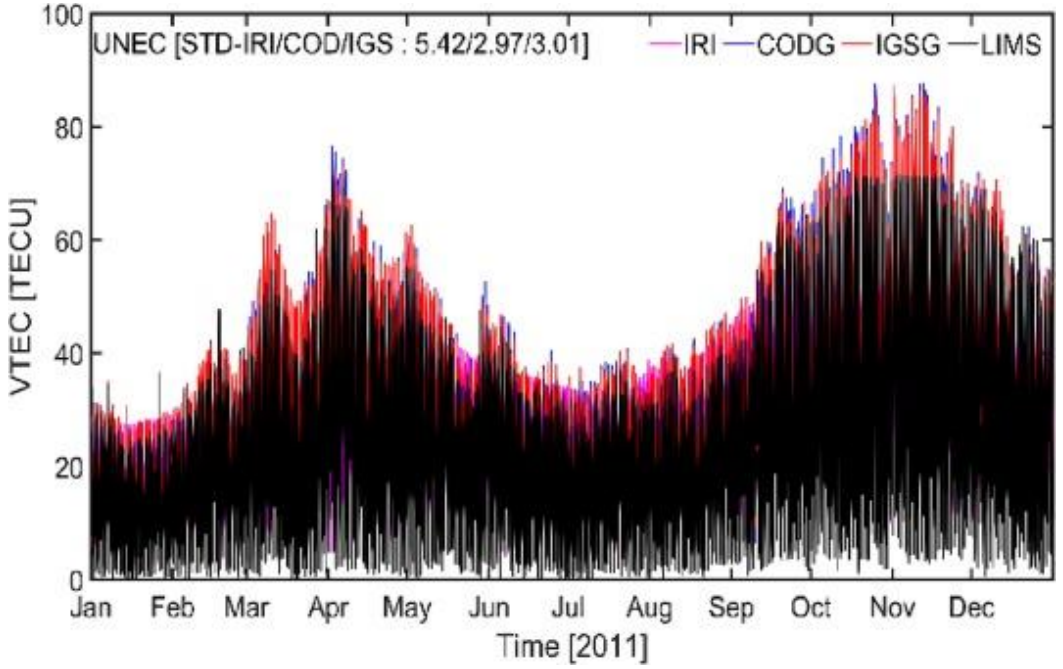
Theory

- **Time series**
  - Sensors collect the observations describing temporal changes of physical phenomena e.g. TEC calculated from GNSS phase data
  - Time series of such observations include meaningful and correlated signal, but also random noise.
  - The signal comprises of complex combination of variations at different frequencies.
  - These frequencies represent responses to different physical factors



Faruna, S. O., Wijaya, D. D., Setyadji, B., Meilano, I., Utama, A. K., & Okoh, D. (2024). Analyzing high temporal-resolution of GNSS-based ionospheric VTEC over Nigeria. *The Egyptian Journal of Remote Sensing and Space Sciences, 27*(2), 317–328. https://doi.org/10.1016/j.ejrs.2024.03.006

Theory

- **The Fourier Transform**
  - Decomposes complex signal into sinusoidal (and cosine) coefficients
  - maps the time-domain x(t) to the frequency domain X(f)
  - Continuous Fourier Transform (CFT) Discrete Fourier Transform (**DFT**)

$$X(f) = \int_{-\infty}^{\infty} x(t) \cdot e^{-i2\pi ft} dt \qquad X_f = \sum_{n=0}^{N-1} x_n W_N^{-fn} \qquad f = 1, 2, \dots, N-1$$

Where f is the frequency and i denotes the imaginary unit, the complex quantity W_N, known as the twiddle factor, is defined by: $W_N = e^{\frac{i2\pi}{N}}$.

  - The complex exponential basis function can be expressed as the real (cosine) and imaginary (sine) components of a signal

$$e^{-i2\pi ft} = \cos(2\pi ft) - i\sin(2\pi ft)$$

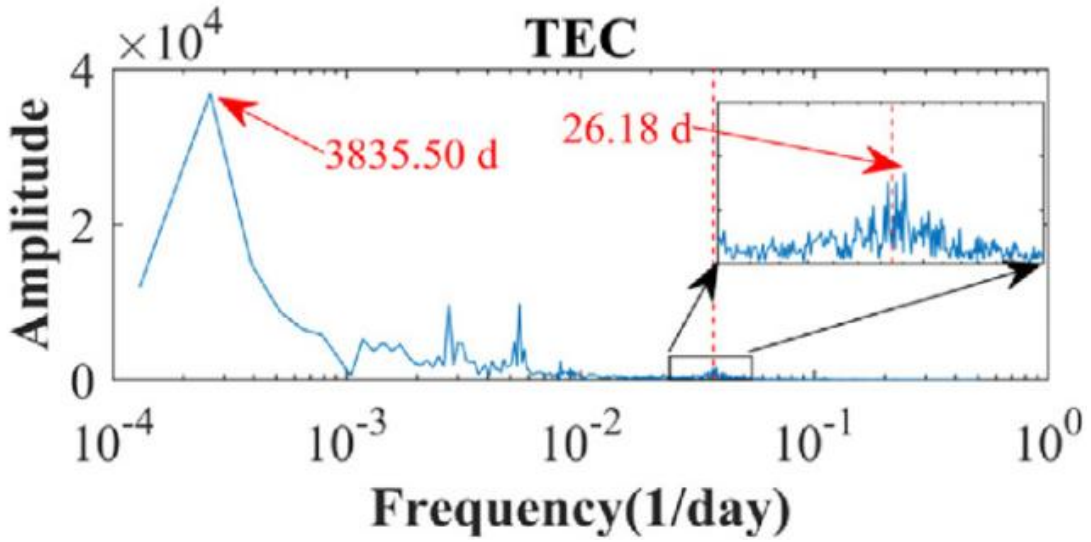
  - The f responses can be also transformed back exactly to xn by the inverse Fourier transform

$$x_n = \frac{1}{N} \sum_{f=0}^{N-1} X_f W_N^{fn} \qquad n = 1, 2, \dots, N-1$$

Theory

- **Power Spectral Density (PSD)**
  - The amplitude of signal responses in the frequency domain determines power spectral density (PSD) at individual frequencies/wavelengths
  - The cumulative PSD can be calculated for the entire length of the analyzed time series, producing one-dimensional power spectra at different frequencies

$$PSD_f = |X_f|^2$$



Feng, J., Y. Zhang, W. Li, B. Han, Z. Zhao, T. Zhang and R. Huang (2023). Analysis of ionospheric TEC response to solar and geomagnetic activities at different solar activity stages, Adv. Space Res., 71, 2225–2239, doi:10.1016/j.asr.2022.10.032.

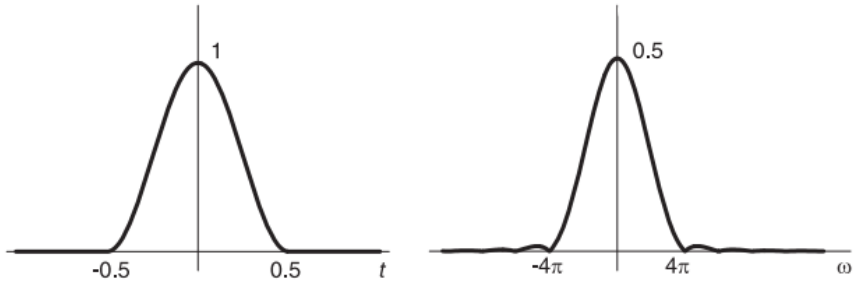
Theory

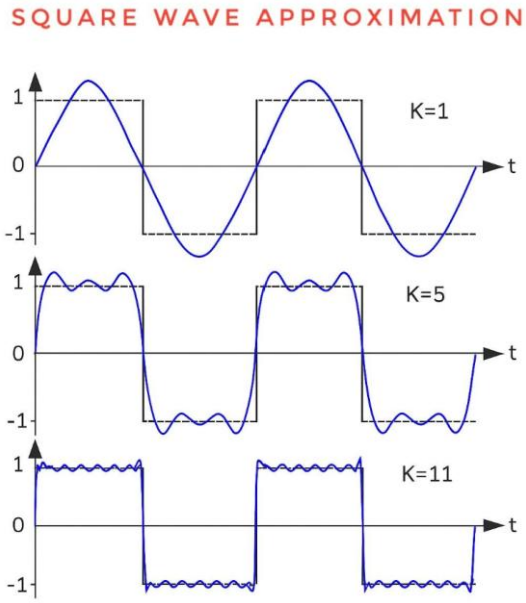
- **Short-Term Fourier Transform and Evolutionary PSD**
  - The evolutionary and windowed STFT is a modification of DFT method, which computes the spectrum of overlapping segments of the time series
  - This results in a two-dimensional representation of the signal

$$X_{f,t} = \sum_{n=0}^{N-1} x_n w_{n-t} W_N^{-fn} \qquad f = 1,2,\dots,N-1$$

where w_n is a discretized window function and t is its time index

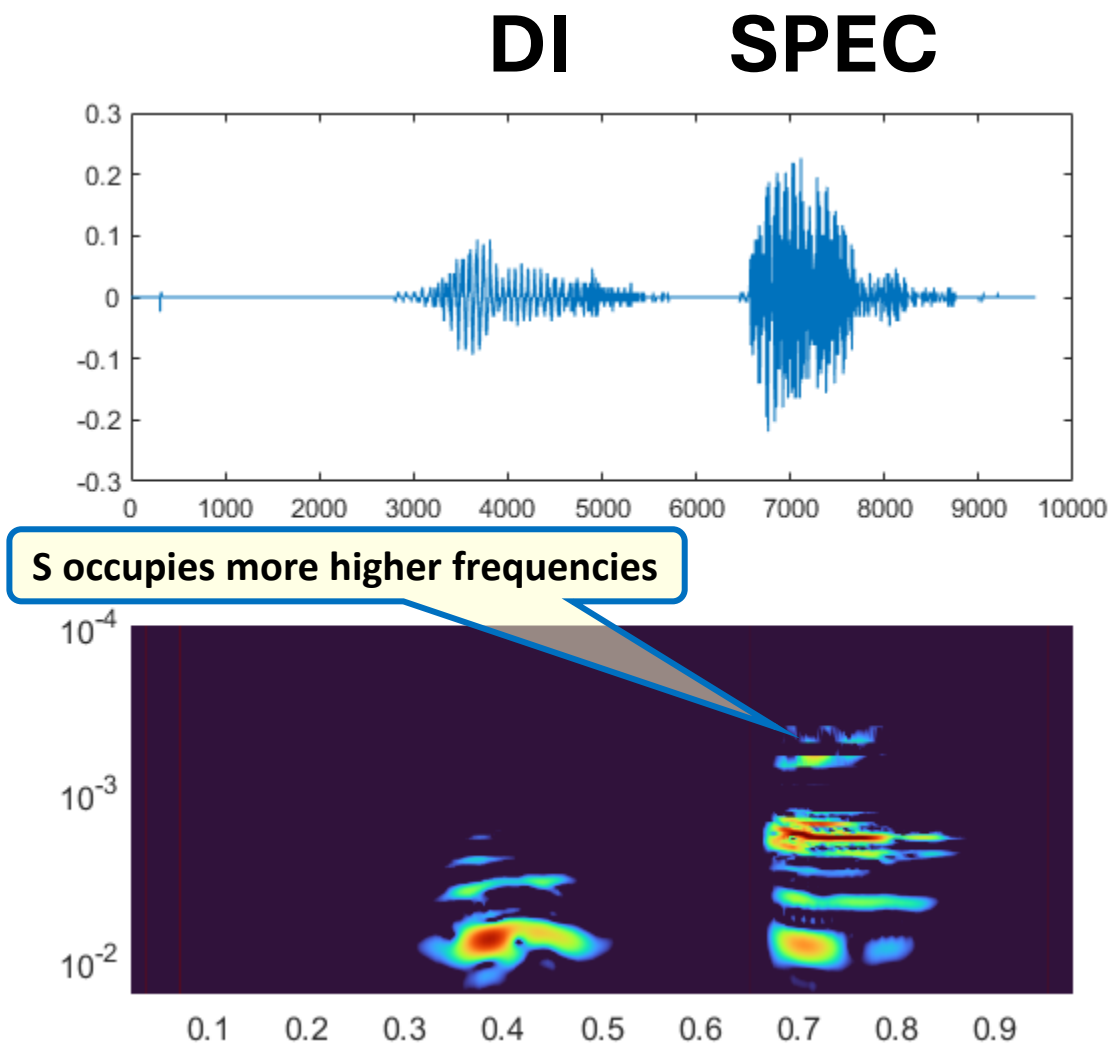
- The frequency response of rectangular window generates an effect called spectral ripples. Therefore, other window shapes are applied in STFT calculations, like e.g. Hanning window





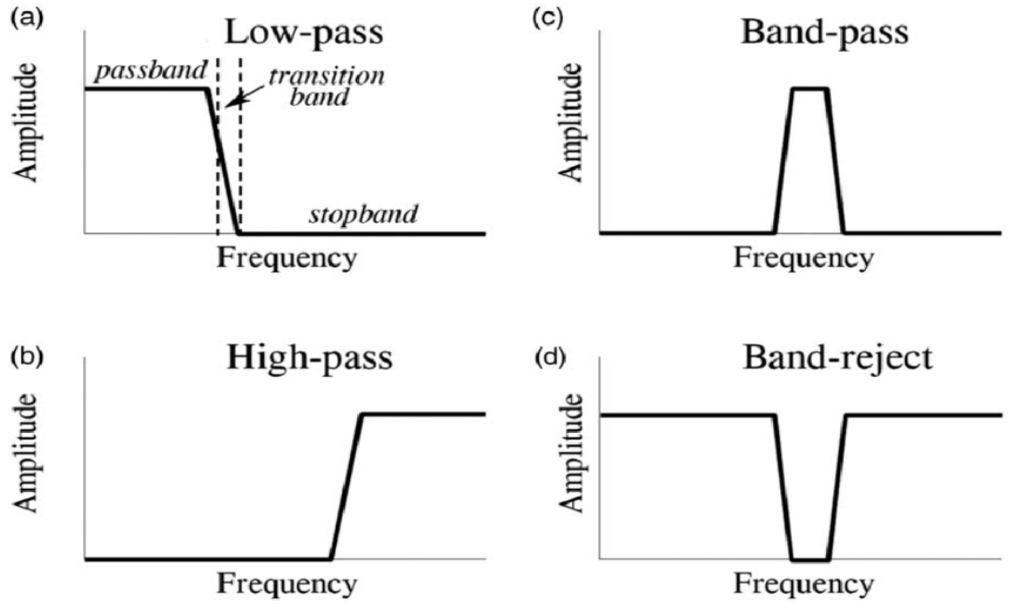
Theory

- **Spectrogram of voice**
  - Aside from analysis of continuous periodicities, STFT detects transient wave-like structures
  - We can call them **periodic signals** or wave-like signals
  - The objective for you is to make **spectrograms of geophysical data**, find specific periodic signals, and find synchronous periodic signals in data of absolutely different origin



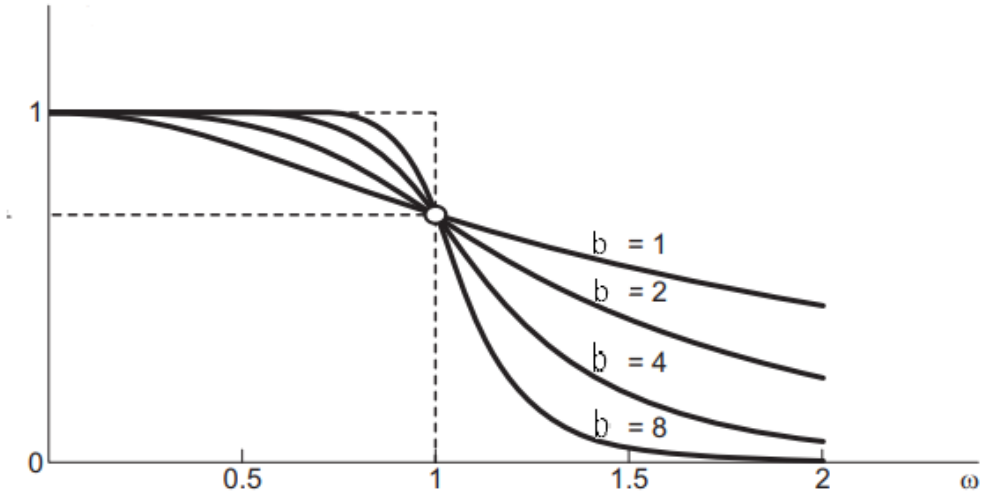
Theory

- **Filtering in the frequency domain**
  - The filtering process is a separation of desired signal frequencies from the entire signal spectrum
  - We can distinguish four filtration processes:
    - high-pass filter keeping the higher frequencies
    - lowpass filter keeping lower frequencies below the selected frequency bound
    - bandpass filter preserving signal between the two frequency bounds
    - band-stop (band-reject) filter removing the selected frequency band



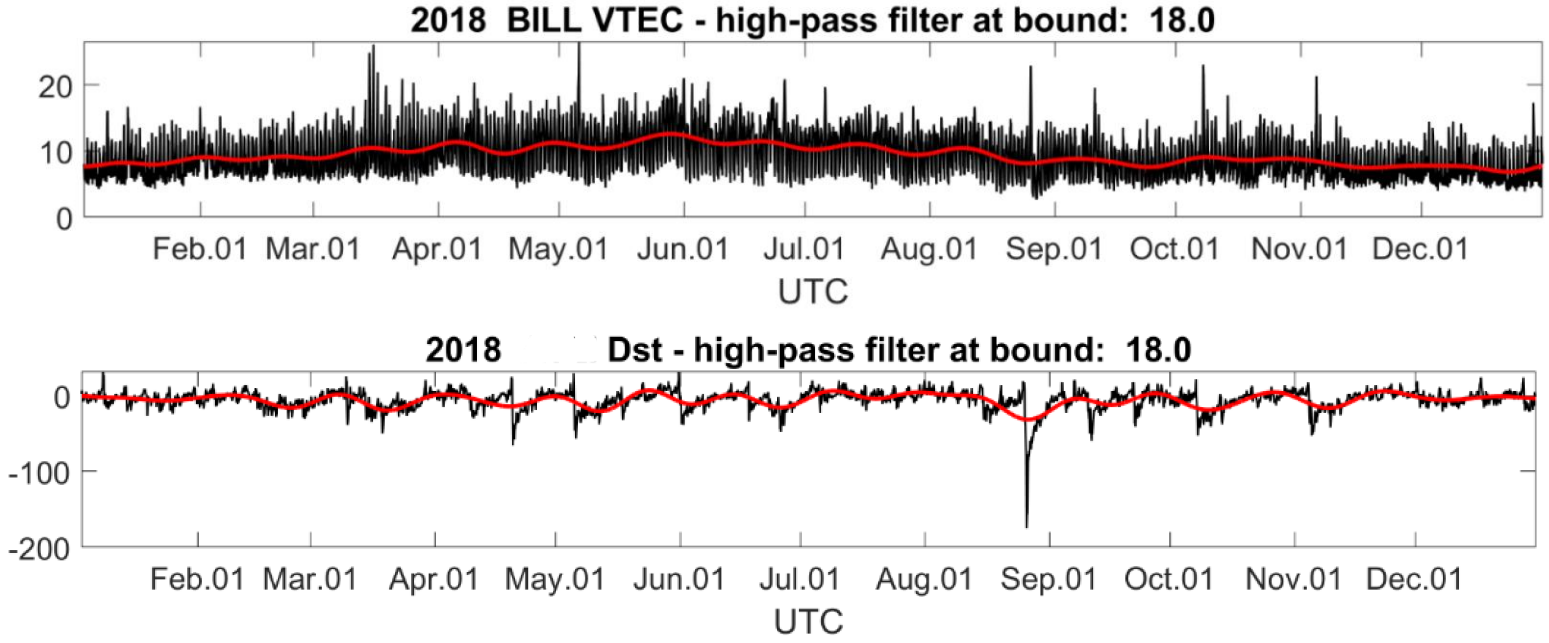
Theory

- **Butterworth filter**
  - Simple rectangular filter can generate the ripples at the filtering bound
  - in practice, a practical lowpass filter applies a transition band around the cutoff frequency
  - This attenuates the ripples around the unity in the passband and around zero in the stopband
  - The frequency response of Butterworth filter is dependent on the shape parameter denoted here as b



Case Study

- **VTEC, Dst, Hp60**
  - The yearly data series of VTEC from phase GNSS data of BILL station (western USA) and Dst index represent the solar minimum of 24th solar cycle. Additionally Hp60 index is attached.



  - The investigated period is characterized by weak and infrequent coronal mass ejection (CME) activity. The moderate solar storms occurring at that time originate mostly from the groups of high-speed streams (HSS) and corotating interaction regions (CIRs).

Link to Practice

- **DFT (filtering) and STFT analyses of data series (VTEC, Kp, Hp60)**

  You will apply DFT (cumulative periodogram), STFT analysis (time-frequency spectrograms) and DFT-based filtering to data series

- **Tools used: (Octave, MATLAB)**

  Please install Octave or Matlab in advance, before the Hands-on Session !!!

  Please install v.11



Thank you for your attention!

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